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J.S. Bach's Unfinished *Contrapunctus XIV* – A Zipf's Law Based Computational Study to Analyze and Classify Attempted Completions

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Abstract

J.S. Bach's *Contrapunctus XIV* from *The Art of Fugue* (*AoF*) remained incomplete at his death, sparking extensive musicological debate: Could Bach have completed it, and the manuscript was subsequently lost? Was the principal theme of the *AoF* intended as the fourth subject in its permutational matrix? Was this fugue even meant for the *AoF*, given the absence of the principal theme in the surviving fragment? [1-6, 12] Beyond scholarly controversy, over 70 attempts have been made to complete this fugue, with widely varying results.

We apply a computational musicology technique based on Zipf's Law to evaluate 20 prominent attempted completions, based on their similarity to Bach's compositional style. Zipf's Law, named after Harvard linguist George Kingsley Zipf, captures scaling (or fractal) properties in natural and human phenomena, including language and music [10, 13]. We have previously developed over 280 musical metrics based on this law [7, 9].

Using **artificial neural networks (ANNs)**, we can successfully distinguish Bach's compositional style from composers like Beethoven and Mozart, with over **90% accuracy**. We can also distinguish stylistic differences within Bach's own works, such as *AoF* and *Well-Tempered Clavier (WTC)* fugues against his Chorales, and even *AoF* against *WTC* fugues. We utilize a high-quality MIDI corpus from the Classical Music Archives. Our results are consistent with earlier studies [7-9, 11, 13].

We observe, based on our results, that certain *Contrapunctus XIV* completions, including those by **Kevin Korsyn, Helmut Walcha, and Geir Øyvind Eskeland**, closely resemble Bach's style. The completions by **Davitt Moroney** and **Indra Nicholas Martindale Hughes** also demonstrate a strong stylistic resemblance, particularly to *Contrapunctus XIV* itself. Also, two of Bach's own *AoF* fugues, *Contrapunctus VI* and *IX*, appear stylistically related to most of the completions. We will discuss additional experiments on more specific stylistic questions.

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