Is C Major Truly Radiant? A Graph-Theoretic Analysis of J.S. Bach's Solo String Works and Their Tonal Characteristics

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Abstract. C major is often perceived as radiant and pure—a perception supported by historical tonal character theories [17, 18] and commonly reported in listening and performance contexts. Recent scholarship suggests that tonal impressions arise from both acoustic properties and cultural conventions [1]. Cognitive science further posits that musical perception reflects not only sound itself, but also its structural organisation [7]. Formal models such as Tymoczko's geometric approach to harmonic space and Ring's transformational analysis of melodic motion provide perspectives on how tonality is structured and perceived [20, 14]. These abstract geometric methodologies provide a powerful framework for addressing the complexity of tonality. However, they do not explicitly model the structural configurations perceived by listeners—such as pitch sequences and intervallic relationships—as identified in cognitive studies.

To examine this perceptual structure, particularly pitch organisation, we apply graph-theoretical analysis, modeling each piece as a network in which nodes represent two-note pitch transitions (2-grams) and edges indicate their sequential adjacency. By modelling perceived elements as nodes, this method offers a framework for linking cognitive interpretation with computational analysis, revealing structural properties. We selected Bach's complete solo string works for their consistent tonality and unaccompanied texture, which reduce tuning-related complexity and likely reflect the composer's sensitivity to resonance and tonal design.

The result shows that works considered as more radiant exhibit higher network heterogeneity (in graph theory terms), with a small number of transitions that occur more frequently than the rest. To explore the underlying causes, we conducted frequency and Pareto analyses of nodes, which revealed that this was associated with symmetry in ascending and descending intervals, reduced diversity in pitch intervals, and a concentration of pitches leading to the tonic [Fig.1]. Notably, being in C major did not necessarily imply greater radiance. Our findings indicate that perceived radiance may emerge from the simplicity and regularity of certain tone sequences.

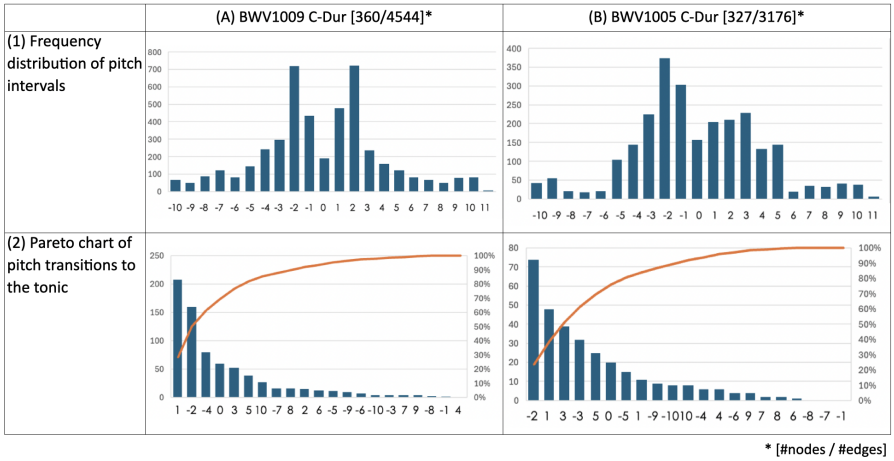

Fig. 1. Two diagrams exploring possible factors contributing to network heterogeneity: (1) a frequency distribution of pitch intervals, and (2) a Pareto chart of transitions to the tonic. The vertical axis shows frequency; the horizontal axis indicates pitch intervals as mod 12 residues, so that intervals differing by octaves (±12) are treated as equivalent. Positive values represent ascending intervals; negative values represent descending. Two contrasting pieces are shown: (A) Cello Suite No. 3 in C major, BWV 1009, and (B) Violin Sonata No. 3 in C major, BWV 1005. (1-A) shows symmetry between ascending and descending intervals; (1-B) is asymmetrical and diverse. (2-A) shows that a few transitions account for most movements toward the tonic; (2-B) displays greater intervallic variety. To help interpret frequency, the number of nodes and edges are indicated as [#Nodes/#Edges].

References

1. Born, G.: Music, Sound and Space: Transformations of Public and Private Experience. Cambridge University Press (2013)
2. Cannas, S., Polo, M.: Euler's "tentamen": Historical and mathematical aspects on the consonance theory. In: Mathematics and Computation in Music. pp. 61–72. Springer International Publishing (2022)
3. Cohn, R.: Introduction to neo-riemannian theory: A survey and a historical perspective. Journal of Music Theory **42**(2), 167–180 (1998)
4. Dahlhaus, C., Gjerdingen, R.O.: Studies on the Origin of Harmonic Tonality. Princeton University Press (1990)
5. Feld, S.: Sound and Sentiment: Birds, Weeping, Poetics, and Song in Kaluli Expression, 3rd edition. Duke University Press (2012)
6. Hevner, K.: The affective character of the major and minor modes in music. The American Journal of Psychology **47**(1), 103–118 (1935)
7. Krumhansl, C.: Rhythm and pitch in music cognition. Psychological bulletin **126**, 159–79 (02 2000). https://doi.org/10.1037//0033-2909.126.1.159
8. Krumhansl, C., Cuddy, L.: A Theory of Tonal Hierarchies in Music, vol. 36, pp. 51–87 (08 2010). https://doi.org/10.1007/978-1-4419-6114-3_3
9. Lehman, B.: Bach's extraordinary temperament: Our rosetta stone: 1. Early Music **33**(1), 3–23 (2005)
10. Lewin, D.: Generalized Musical Intervals and Transformations. Oxford University Press (05 2007). https://doi.org/10.1093/acprof:oso/9780195317138.001.0001
11. Miller, J., Nicosia, V., Sandler, M.: Discovering common practice: Using graph theory to compare harmonic sequences in musical audio collections. p. 93–97. DLfM '21, Association for Computing Machinery (2021)
12. Rameau, J.P.: Treatise on Harmony Reduced to Its Natural Principles. Dover Publications; Reprint edition (June 1, 1971) (1722)
13. Riemann, H.: Ideas for a study "on the imagination of tone". Journal of Music Theory **36**(1), 81–117 (1992)
14. Rings, S.: Tonality and transformation. Oxford studies in music theory, Oxford University Press, New York (2011)
15. Roederer, J.G.: Superposition and Successions of Complex Tones and the Integral Perception of Music, pp. 167–198. Springer US, New York, NY (2009)
16. Schneider, A.: Systematic musicology: A historical interdisciplinary perspective. In: Springer Handbook of Systematic Musicology (2018)
17. Schubart, D.C.F.: Ideen zu einer Aesthetik der Tonkunst. Legare Street Press (July 18, 2023) (1806)
18. Steblin, R.: A history of key characteristics in the eighteenth and early nineteenth centuries. University of Rochester Press, Rochester, N.Y, 2nd ed. edn. (2002)
19. Tymoczko, D.: The geometry of musical chords. Science **313**(5783), 72–74 (2006)
20. Tymoczko, D.: A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice. Oxford University Press (2011)
21. Tymoczko, D.: Why topology? Journal of Mathematics and Music **14**(2), 114–169 (2020)
22. Willimek, D., Willimek, B.: Revealing the mystery of emotions in sounds. Auditory Perception & Cognition **6**(1-2), 128–153 (2023)
23. Young, J.O.: Key, temperament and musical expression. The Journal of Aesthetics and Art Criticism **49**(3), 235–242 (1991)
24. Yust, J.: Schubert's harmonic language and fourier phase space. Journal of Music Theory **59**(1), 121–181 (2015)