A Hybrid Framework for Chord Analysis: Lambek Calculus and Tonal Pitch Space

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Abstract

Lambek Calculus, a logical system originally developed for categorial grammar, has been proposed as a grammatical framework for analyzing chord sequences in (Bizzarri and Tojo 2024, 2025). This formalism is advantageous because it allows harmonic relationships to be represented through rigorous syntactic derivations, enabling the application of methods from formal logic and linguistic analysis to music theory. Therefore, we adopted multiple accessibilities and we could simulate the modulation with it. A recent work (Bizzarri and Tojo 2025) introduced methods to evaluate the complexity of chord sequences by measuring the depth of derivations and the number of modulations within LC proofs. However, while logically well-grounded, these measures may oversimplify the musical richness of harmonic progressions.

To address this issue, a statistical perspective based on the Tonal Pitch Space (TPS), i.e., a model that assigns numerical distances between chords, has been incorporated. Since TPS provides fixed values, a learning system has been developed to statistically evaluate and refine these distances using a corpus of chord sequences. As chords are explicitly interpreted within specific tonalities, it becomes necessary to incorporate the notion of context, i.e., a possible world interpreted as a musical region. However, Lambek Calculus lacks native labels corresponding to such possible worlds.

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