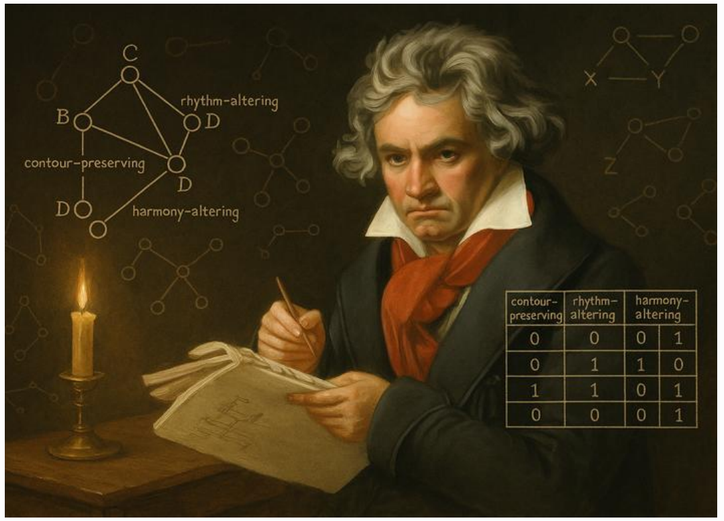

Probabilistic Multilabel Graphical Modelling of Motif Transformations in Symbolic Music

Third International Conference on Computational and Cognitive Musicology

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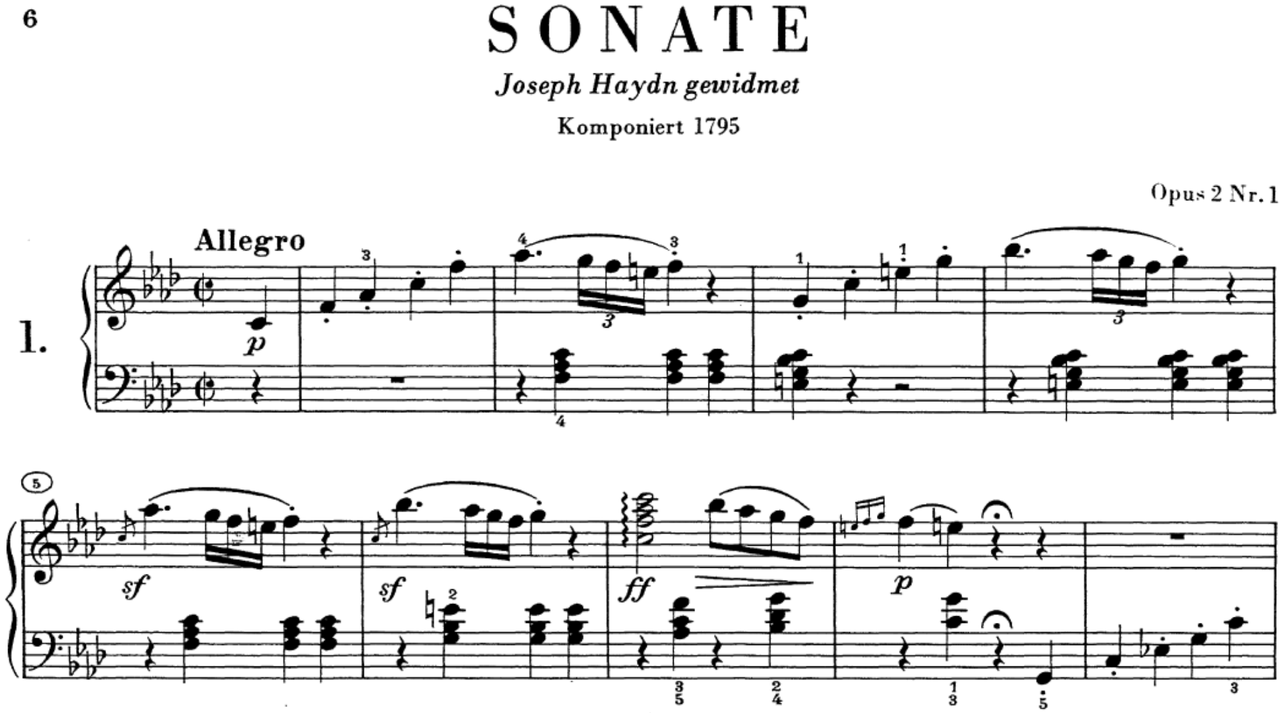
Goal

- Build probabilistic framework for motif transformations.
- Model the stylistic patterns in classical symbolic music.
- Provide interpretable, distributional inference across Beethoven's periods.
- Learn effects of features & co-occurrence patterns.



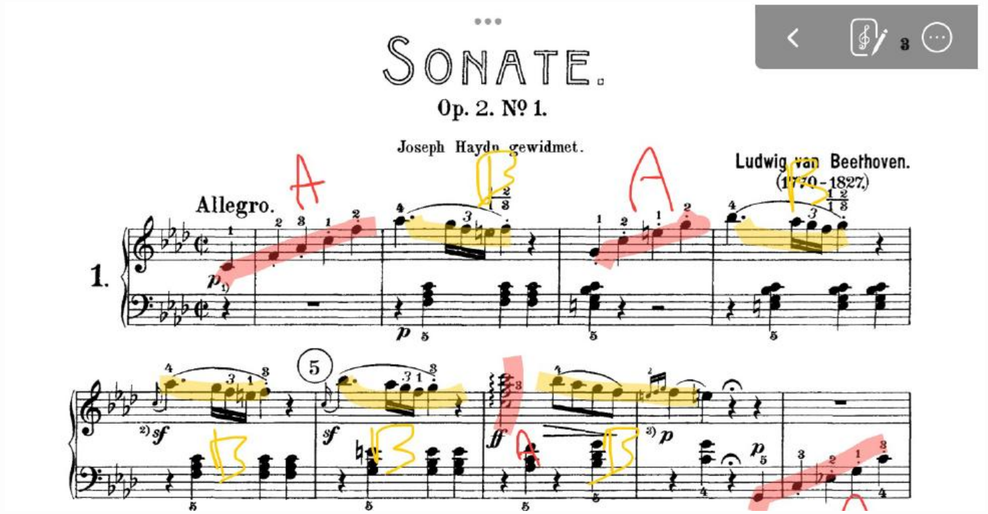
Motivation

Some building blocks

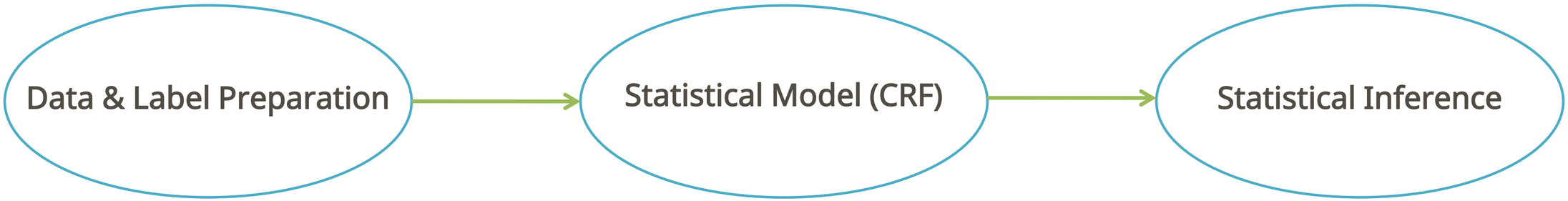
- Motif - a short, recognizable musical idea that recurs (sometimes with variation).
- Motif transformations - when motifs reappear with variation (musical modification, not a change of domain).
- Segment - a local passage where motifs interact.

Why study motifs?

- Integrating interpretability into structural analysis of motif transformations is still relatively underexplored.
- New symbolic corpora (BPSD, BPS-Motif, BPS-FH) enable systematic study.

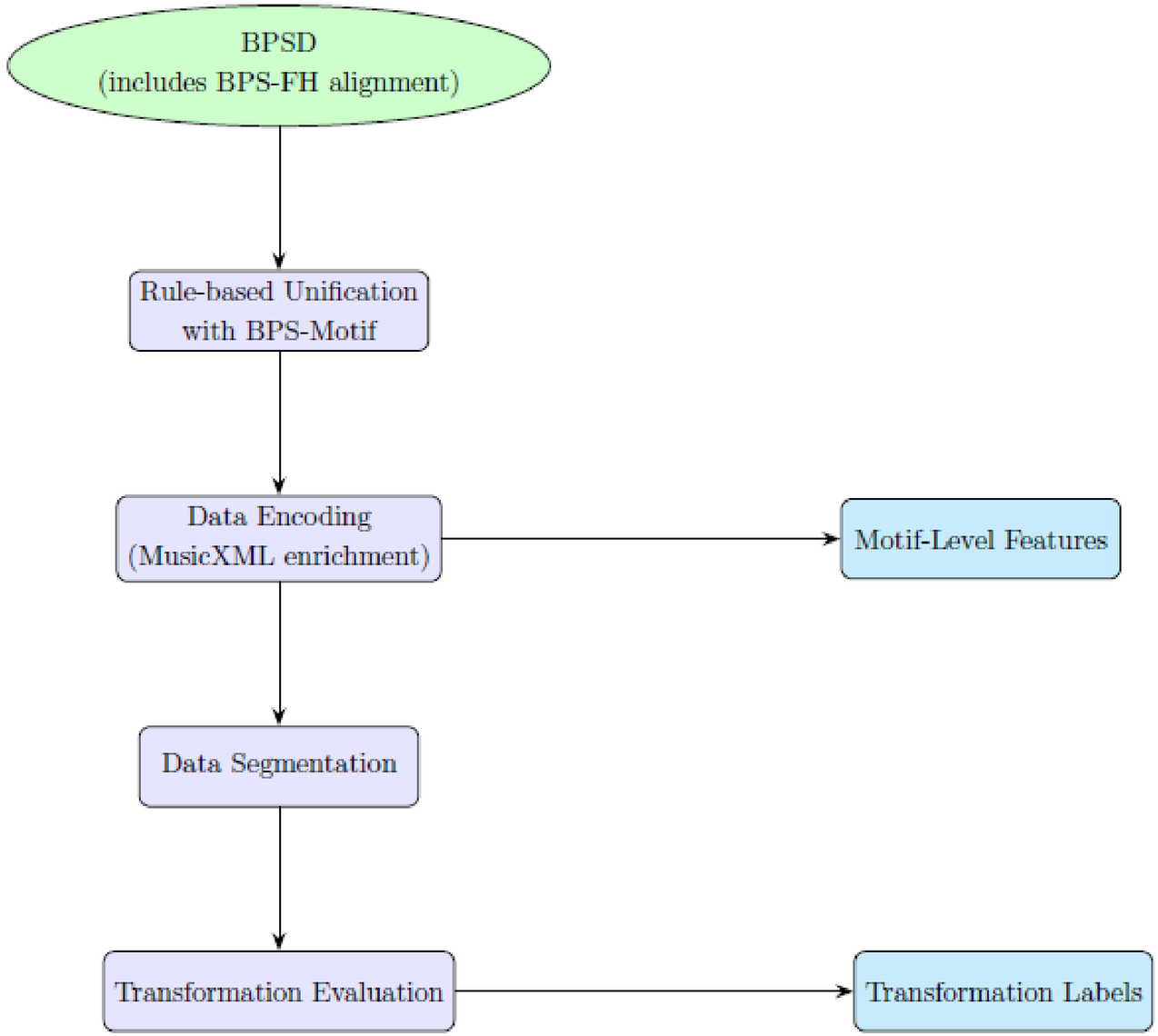


Analytical Framework



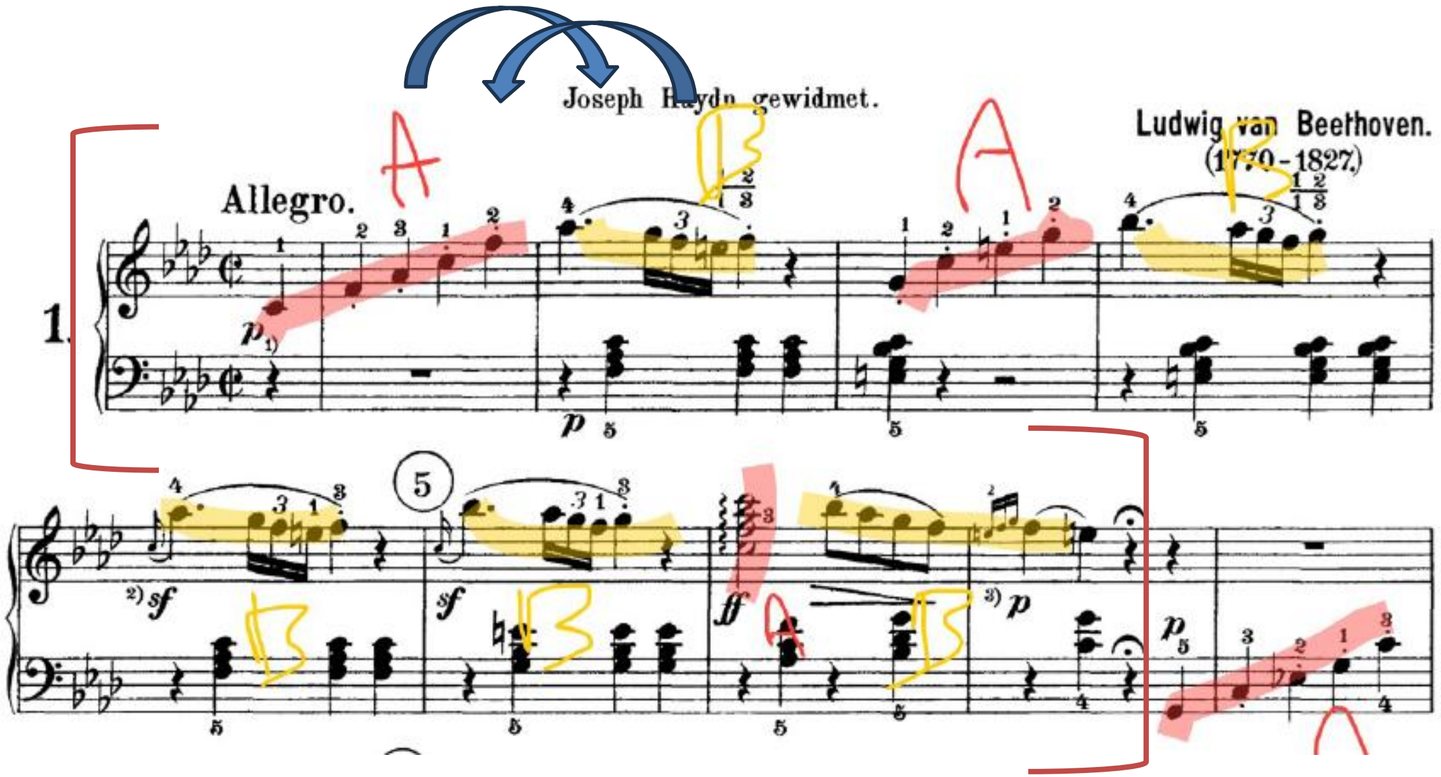
Data

Data Process Pipeline



Data

General Structure



- Motif instances (A, B, …) influence each other through proximity.
- First instance in each group is often perceived as salient (reference point).
- We segment pieces using 3 rules: silence, repeated pitch, harmonic criteria.

Segment Boundary

Within each segment:

- Each motif type has an anchor (reference).
- Dependencies between motif instances can be evaluated locally.

Data

Motif-Level Features

Feature	Description	Category
Spread Harmonic Complexity	Difference between maximum and minimum harmonic complexity values across chords in the motif instance.	Harmonic
Secondary Chord Proportion	Proportion of secondary chords relative to all chords in the motif instance.	Harmonic
Key Change Count	Number of local key changes normalized by motif duration.	Harmonic
Pitch Spread	Range (max–min) of MIDI pitch values.	Melodic
Motif Pitch Register	Median relative pitch height (normalized by key center).	Melodic
Inter-Onset Interval (IOI) SD	Standard deviation of standardized inter-onset intervals.	Rhythmic
Silence Proportion	Proportion of silence (gaps between notes) relative to motif duration.	Rhythmic
Metrical Stress Rate	Average metrical stress weight based on the time signature and beat position.	Rhythmic
Expressive Density	Mean density of expressive markings (slurs, cresc., dim., accents).	Expressive
Accentuation SD	Standard deviation of dynamic levels across the motif instance.	Expressive

Data

Transformation Families

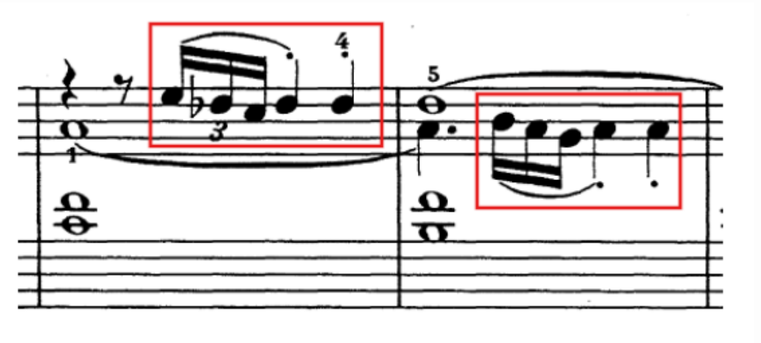
Identity
Motif is identical to its anchor in length, pitch, rhythm, harmony, and order.

Contour-Preserving
The direction of pitch intervals is preserved (e.g., up/down pattern), and actual intervals differ.

Salient Leap-Preserving
Large leaps (based on second derivative of pitch) are preserved, maintaining prominent jumps.

Rhythm-Preserving
The motif maintains the rhythmic structure when the differences between their inter-onset intervals are consistent, as quantified by low mean absolute deviation from the median.

Symmetry-Preserving
The motif is an inversion, retrograde, or both (inverted retrograde), relative to the anchor, using the first note as the axis of inversion; allows “imperfect inversions” (e.g., fourths ↔ fifths, major ↔ minor).



Example: Harmony- (IV^{65} & ii^7 ⇒ PD) & intervallic-preserving

Note Addition/Removal
Notes (which are not duplicated pitches) are inserted or removed.

Harmony-Preserving
The harmonic function zones (Tonic, Pre-Dominant, Dominant) remain consistent across aligned notes.

Intervalic-Preserving
The interval roles (e.g., third, fifth, tritone) between notes are preserved modulo 12, even if absolute pitches differ.

Data

Illustration

$X =$

Motif	Pitch Spread Feature
m_1^{A1}	$\max(\text{pitch}(m_1^{A1})) - \min(\text{pitch}(m_1^{A1}))$
m_1^{B1}	...
m_1^{A2}	...
m_1^{B2}	...
m_1^{B3}	...
m_1^{B4}	...
m_1^{A3}	...
m_1^{B5}	...

$Y =$

Motif	Identity	Contour	Note Edit	Symmetry
m_1^{A1}	1	0	0	0
m_1^{B1}	1	0	0	0
m_1^{A2}	0	1	1	1
m_1^{B2}	...	...	...	...
m_1^{B3}	...	...	...	...
m_1^{B4}	...	...	...	...
m_1^{A3}	...	...	...	...
m_1^{B5}	...	...	...	...

Model

GRAPHICAL MODELING

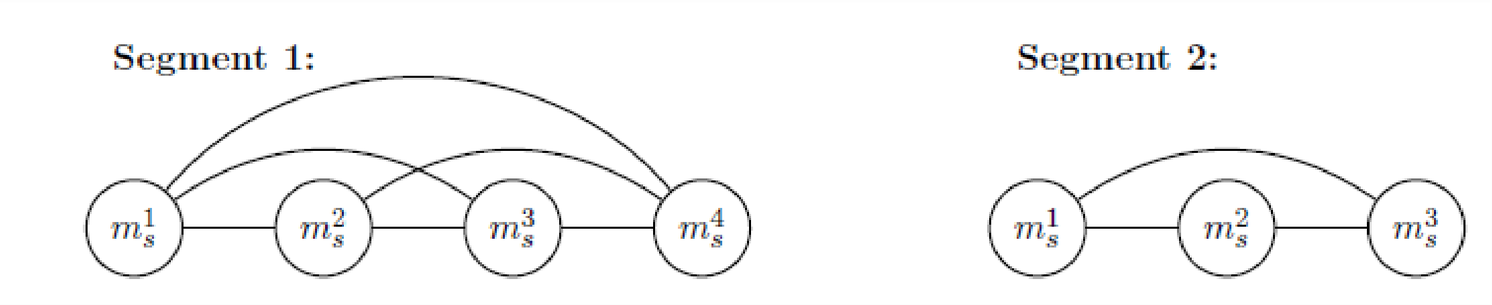
Graph Definition:
For each sonata, we construct a block-diagonal graph $G = (V, E)$, where nodes represent motif instances and edges represent proximity, each block corresponds to a segment, and no connections exist between segments.

Node Representation:
Each node m_s^j (motif instance j in segment s) corresponds to a motif instance and is labeled by a binary vector of transformation families $Y\left(m_s^j\right) \in \{0,1\}^Q$.

Edge Construction:
Within each segment, motifs are fully connected, with edge weights given by a Gaussian decay of their onset distance:

$$w\left(m_s^i, m_s^j\right) = \begin{cases} exp\left(-\frac{(j-i)^2}{\sigma^2}\right) \; if \; exp\left(-\frac{(j-i)^2}{\sigma^2}\right) < 10^{-5} \\ 0 \quad o.w \end{cases}$$

Example:



Model

Conditional Random Fields

Goal: Model dependencies among motif transformation labels (and covariates) within each segment.
Nodes: Motif instances (Motif level feature matrix $X \in R^{N \times D}$ (N motif instances, D motif level features)
Edges: Instances proximity under a corresponding segment.
Labels: Binary matrix $Y \in \{0,1\}^{N \times Q}$ indicating transformation families.
Graph: Adjacency matrix $\mathcal{A}$ encodes proximity within segments (block-diagonal).

CRF Equation:

$$P(\mathbf{Y} \mid \mathbf{X}) = \frac{1}{Z(\mathbf{X})} \exp\left(\underbrace{\sum_i \sum_q Y_i^q \, X_i^\top \alpha_{\cdot q}}_{\text{Unary: features} \to \text{labels}} + \underbrace{\sum_{i,j} \mathcal{A}_{ij} \sum_{q,r} Y_i^q Y_j^r \, \beta_{qr}}_{\text{Pairwise: label dependencies}} \right)$$

Unary terms (α): effect of motif-level features on label activations.
Pairwise terms (β): capture co-occurrence of transformation families across connected motifs.
Z(X): partition function (normalization).

Objective function:

$$\mathcal{J}_{\text{PL,reg}} = \sum_{i=1}^{N} \sum_{q=1}^{Q} \left[\log(1 + e^{z_{iq}}) - Y_i^q z_{iq} \right] + \lambda_\alpha \|\alpha\|_2^2 + \lambda_\beta \|\beta\|_2^2$$

Without the pairwise terms → the model reduces to multilabel logistic regression!

Statistical Inference

CLR Statistic:

$$CLR = 2\left(\mathcal{L}_{comp}(\hat{\theta}) - \mathcal{L}_{comp}(\hat{\theta}_0)\right)$$

- Compares full vs. restricted model (drop a block of parameters).
- Assesses if a group of features or pairwise terms significantly improves model fit.
- Used for block-level inference.

Comparisons Tested: Intercept → Unary (α), Unary → Full ($\alpha + \beta$), Pairwise → Full.

Wald CI:

$$\hat{\theta}_j \pm z_{1-\frac{\alpha}{2}} \cdot \sqrt{\left[\widehat{Var}(\hat{\theta})\right]_{jj}}$$

- Tests whether an individual coefficient contributes significantly.
- Provides interpretable effect size + uncertainty.
- Adjusted with Benjamini–Hochberg (FDR, $q = 0.1$) to control false discoveries.

Period-Based Models & Effective Sample Size

Period Based:
Beethoven sonatas divided chronologically, close to the Early, Middle, Late.
Each period estimated separately to capture stylistic variation.

Periods:
- 1795-1800 (Nos. 1–11, 19–20): Mozart/Haydn influence.
- 1801-1805 (Nos. 12–18, 21–25): “Heroic” style.
- 1810-1822 (Nos. 26–32): Experimental (with some bridge between the heroic period and the late period).

Effective Sample Size (ESS)
The independent unit is the segment (not the motif instance), as pairwise dependencies rely on within-segment proximity. This matches cluster-robust inference (Cameron & Miller, 2015).
- Unary block: ESS = number of segments.
- Pairwise block: ESS for β_{pq} = number of segments where p or q appear.

Transformation Frequencies

Label	Count	Frequency
identity	502	0.257
contour	307	0.157
salient_leap	447	0.229
rhythm	263	0.135
symmetry	178	0.091
note_edit	757	0.388
harmony	215	0.110
intervalic	377	0.193

Early Period: 1,953 motif instances.

Label	Count	Frequency
identity	426	0.227
contour	274	0.146
salient_leap	425	0.235
rhythm	235	0.125
symmetry	185	0.099
note_edit	731	0.390
harmony	216	0.115
intervalic	394	0.210

Middle Period: 1,874 motif instances.

Label	Count	Frequency
identity	174	0.195
contour	157	0.176
salient_leap	228	0.255
rhythm	220	0.246
symmetry	86	0.096
note_edit	378	0.423
harmony	98	0.110
intervalic	168	0.118

Late Period: 893 motif instances.

CLR Results

Test	Obs CLR	p^{perm}
Null → Unary	549.220	0.001
Unary → Full	907.830	0.004
Pairwise → Full	333.560	0.001

Early Period: Permutation CLR (1,000 resamples)

Test	Obs CLR	p^{perm}
Null → Unary	383.908	0.001
Unary → Full	964.599	0.015
Pairwise → Full	267.747	0.001

Middle Period: Permutation CLR (1,000 resamples)

Test	Obs CLR	p^{perm}
Null → Unary	399.899	0.001
Unary → Full	611.384	0.001
Pairwise → Full	229.666	0.001

Late Period: Permutation CLR (1,000 resamples)

Inference

Results Summary

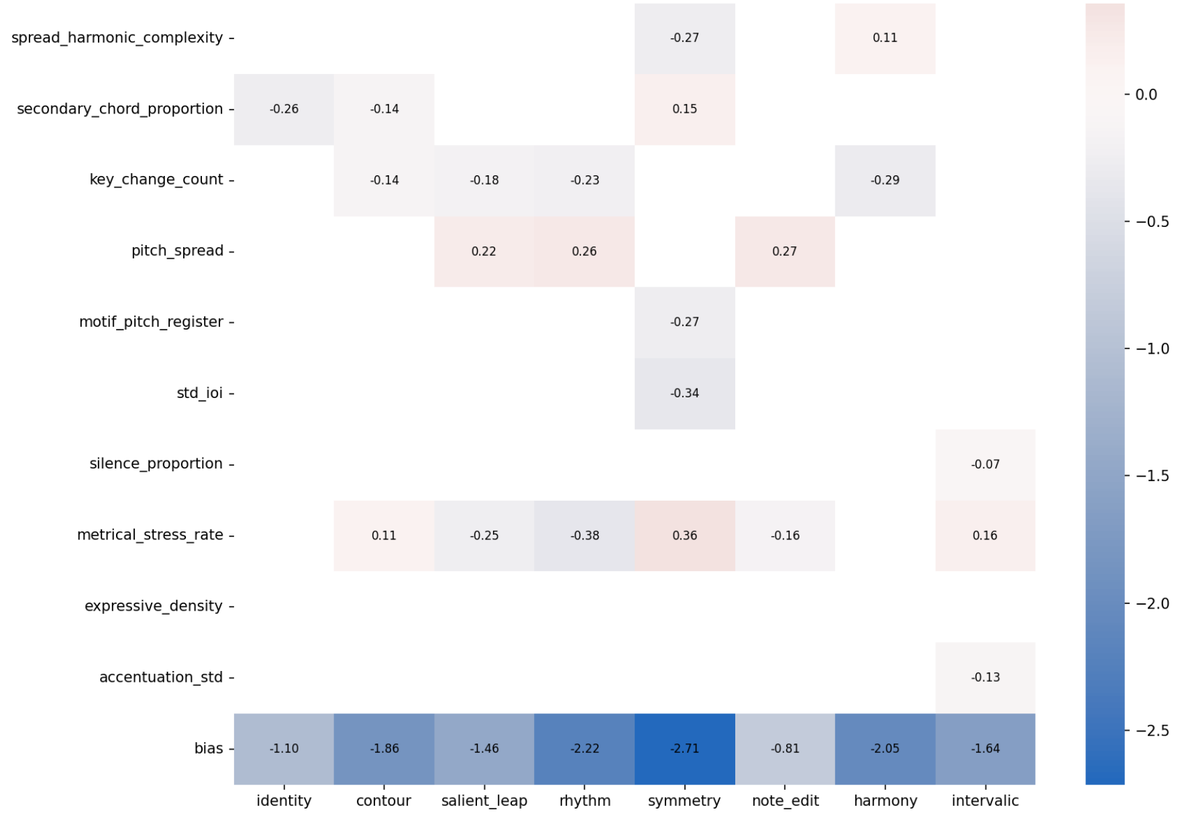
Label	Parameter	Estimate (SE)	90% CI	q^{BH}
Identity	Bias	-1.103*** (0.104)	[-1.275, -0.931]	0.000
Identity	Secondary Chord Proportion	-0.256*** (0.050)	[-0.339, -0.173]	0.000
Contour	Bias	-1.858*** (0.082)	[-1.992, -1.723]	0.000
Contour	Key Change Count	-0.135*** (0.036)	[-0.194, -0.077]	0.001
Contour	Metrical Stress Rate	0.108* (0.042)	[0.038, 0.177]	0.034
Contour	Secondary Chord Proportion	-0.135** (0.039)	[-0.200, -0.071]	0.002
Salient leap	Bias	-1.457*** (0.105)	[-1.630, -1.284]	0.000
Salient leap	Key Change Count	-0.177* (0.064)	[-0.281, -0.072]	0.020
Salient leap	Metrical Stress Rate	-0.248*** (0.044)	[-0.321, -0.175]	0.000
Salient leap	Pitch Spread	0.219** (0.062)	[0.117, 0.322]	0.002
Rhythm	Bias	-2.221*** (0.100)	[-2.386, -2.057]	0.000
Rhythm	Key Change Count	-0.231*** (0.050)	[-0.313, -0.150]	0.000
Rhythm	Metrical Stress Rate	-0.377*** (0.040)	[-0.442, -0.311]	0.000
Rhythm	Pitch Spread	0.263*** (0.065)	[0.156, 0.369]	0.000
Symmetry	Bias	-2.714*** (0.062)	[-2.815, -2.613]	0.000
Symmetry	Metrical Stress Rate	0.357*** (0.038)	[0.294, 0.421]	0.000
Symmetry	Motif Pitch Register	-0.275* (0.091)	[-0.424, -0.126]	0.009
Symmetry	SD IOI	-0.340*** (0.082)	[-0.474, -0.205]	0.000
Symmetry	Secondary Chord Proportion	0.147* (0.067)	[0.037, 0.258]	0.083
Symmetry	Spread Harmonic Complexity	-0.268*** (0.044)	[-0.341, -0.195]	0.000
Note edit	Bias	-0.810*** (0.085)	[-0.950, -0.671]	0.000
Note edit	Metrical Stress Rate	-0.160** (0.049)	[-0.240, -0.080]	0.004
Note edit	Pitch Spread	0.270* (0.103)	[0.100, 0.440]	0.030
Harmony	Bias	-2.050*** (0.065)	[-2.157, -1.944]	0.000
Harmony	Key Change Count	-0.293*** (0.035)	[-0.351, -0.235]	0.000
Harmony	Spread Harmonic Complexity	0.113* (0.042)	[0.043, 0.183]	0.027
Intervalic	Accentuation SD	-0.131* (0.056)	[-0.224, -0.039]	0.060
Intervalic	Bias	-1.636*** (0.100)	[-1.801, -1.471]	0.000
Intervalic	Metrical Stress Rate	0.159* (0.051)	[0.075, 0.243]	0.007
Intervalic	Silence Proportion	-0.067* (0.029)	[-0.115, -0.020]	0.059

Early Period: significant $\hat{\alpha}$ parameters

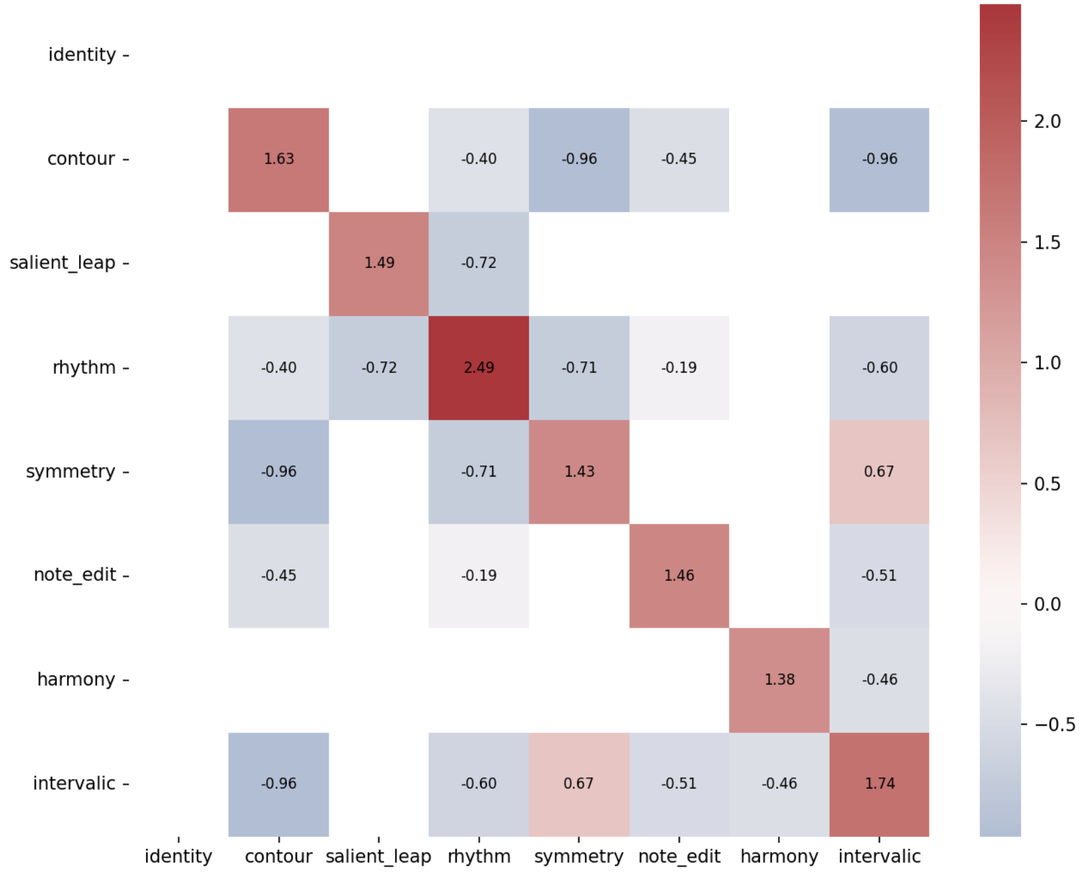
Pair	Estimate (SE)	90% CI	q^{BH}
Contour–Contour	1.630*** (0.329)	[1.089, 2.171]	0.000
Contour–Rhythm	-0.400** (0.129)	[-0.612, -0.188]	0.004
Contour–Symmetry	-0.960*** (0.181)	[-1.258, -0.662]	0.000
Contour–Note edit	-0.453*** (0.079)	[-0.582, -0.323]	0.000
Contour–Intervalic	-0.965*** (0.127)	[-1.174, -0.756]	0.000
Salient leap–Salient leap	1.491*** (0.321)	[0.963, 2.020]	0.000
Salient leap–Rhythm	-0.719*** (0.175)	[-1.008, -0.431]	0.000
Rhythm–Rhythm	2.488*** (0.436)	[1.771, 3.205]	0.000
Rhythm–Symmetry	-0.715*** (0.171)	[-0.995, -0.434]	0.000
Rhythm–Note edit	-0.189* (0.082)	[-0.324, -0.055]	0.041
Rhythm–Intervalic	-0.603*** (0.076)	[-0.728, -0.478]	0.000
Symmetry–Symmetry	1.434* (0.557)	[0.519, 2.350]	0.021
Symmetry–Intervalic	0.671*** (0.181)	[0.373, 0.969]	0.001
Note edit–Note edit	1.459*** (0.282)	[0.995, 1.923]	0.000
Note edit–Intervalic	-0.510** (0.151)	[-0.759, -0.261]	0.002
Harmony–Harmony	1.381*** (0.379)	[0.759, 2.004]	0.001
Harmony–Intervalic	-0.462*** (0.081)	[-0.595, -0.329]	0.000
Intervalic–Intervalic	1.738*** (0.235)	[1.351, 2.125]	0.000

Early Period: significant $\hat{\beta}$ parameters

Inference



Early Period: significant $\hat{\alpha}$ parameters



Early Period: significant $\hat{\beta}$ parameters

Inference

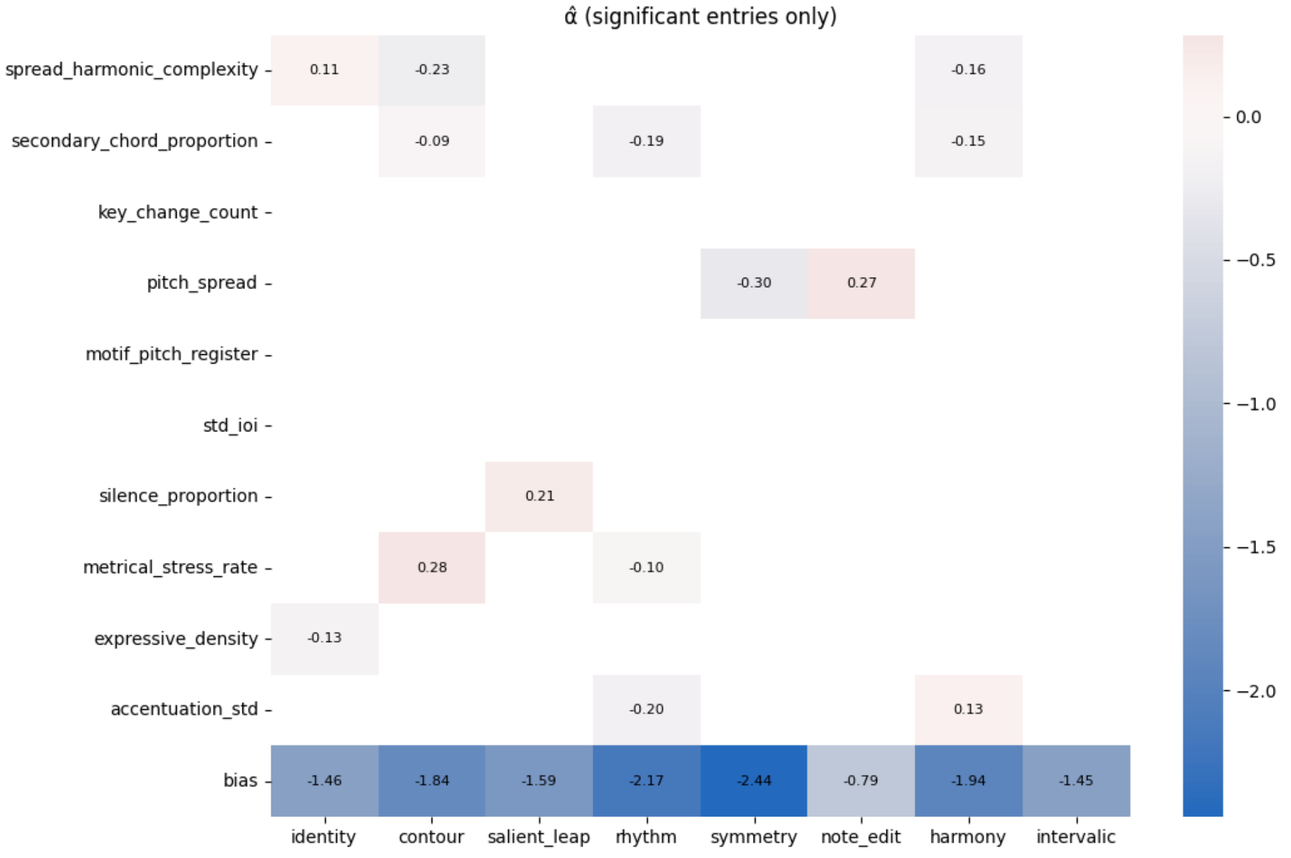
Label	Parameter	Estimate (SE)	90% CI	q^{BH}
Identity	Bias	-1.458*** (0.076)	[-1.583, -1.333]	0.000
Identity	Expressive Density	-0.133** (0.037)	[-0.193, -0.072]	0.002
Identity	Spread Harmonic Complexity	0.105* (0.047)	[0.028, 0.182]	0.096
Contour	Bias	-1.841*** (0.070)	[-1.956, -1.726]	0.000
Contour	Metrical Stress Rate	0.283*** (0.061)	[0.182, 0.383]	0.000
Contour	Secondary Chord Proportion	-0.086* (0.028)	[-0.132, -0.039]	0.014
Contour	Spread Harmonic Complexity	-0.233** (0.061)	[-0.334, -0.132]	0.001
Salient leap	Bias	-1.594*** (0.127)	[-1.803, -1.384]	0.000
Salient leap	Silence Proportion	0.206* (0.076)	[0.081, 0.331]	0.033
Rhythm	Accentuation SD	-0.196** (0.052)	[-0.281, -0.110]	0.001
Rhythm	Bias	-2.172*** (0.057)	[-2.266, -2.079]	0.000
Rhythm	Metrical Stress Rate	-0.102* (0.036)	[-0.161, -0.044]	0.021
Rhythm	Secondary Chord Proportion	-0.190* (0.080)	[-0.322, -0.057]	0.077
Symmetry	Bias	-2.440*** (0.079)	[-2.570, -2.311]	0.000
Symmetry	Pitch Spread	-0.304*** (0.051)	[-0.389, -0.220]	0.000
Note edit	Bias	-0.785*** (0.123)	[-0.988, -0.583]	0.000
Note edit	Pitch Spread	0.270* (0.114)	[0.082, 0.457]	0.077
Harmony	Accentuation SD	0.131* (0.043)	[0.061, 0.202]	0.012
Harmony	Bias	-1.936*** (0.057)	[-2.031, -1.842]	0.000
Harmony	Secondary Chord Proportion	-0.148* (0.056)	[-0.240, -0.055]	0.040
Harmony	Spread Harmonic Complexity	-0.163** (0.046)	[-0.238, -0.087]	0.002
Intervalic	Bias	-1.454*** (0.108)	[-1.633, -1.276]	0.000

Middle Period: significant $\hat{\alpha}$ parameters

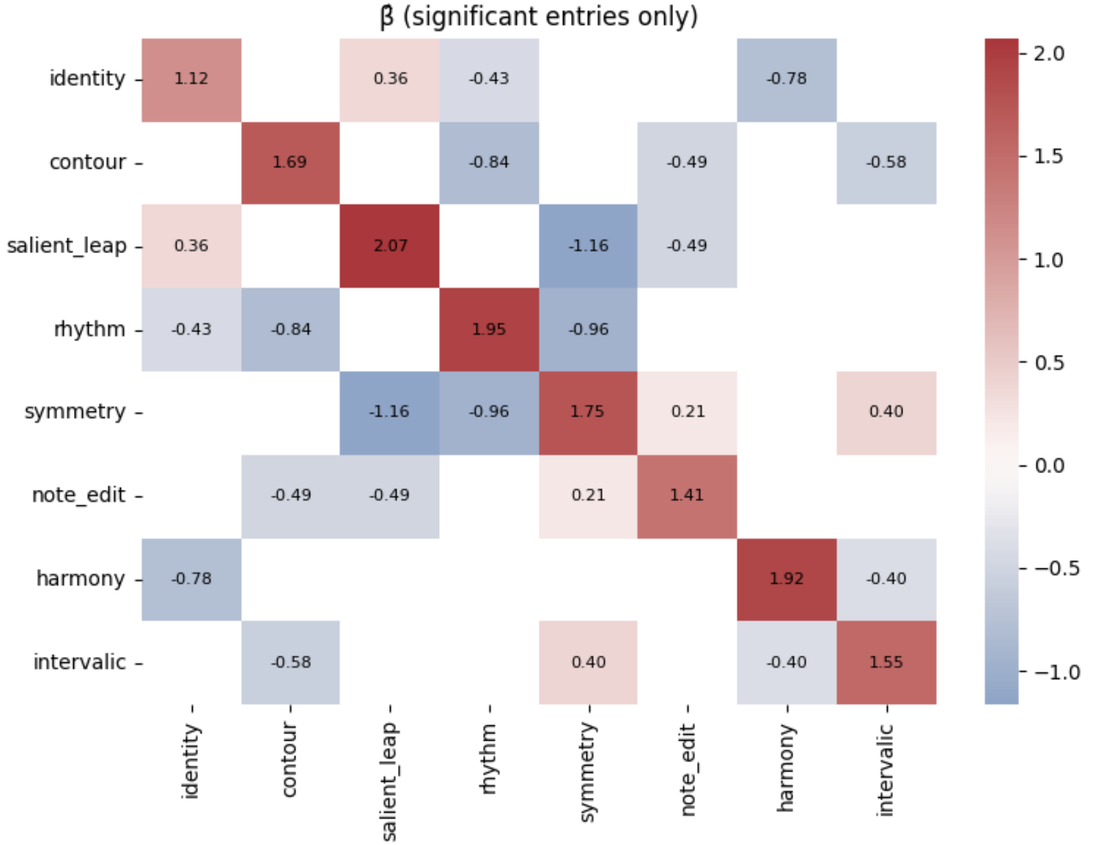
Pair	Estimate (SE)	90% CI	q^{BH}
Identity–Identity	1.122* (0.366)	[0.521, 1.723]	0.005
Identity–Salient leap	0.356* (0.168)	[0.080, 0.632]	0.061
Identity–Rhythm	-0.426* (0.138)	[-0.653, -0.199]	0.005
Identity–Harmony	-0.778*** (0.201)	[-1.109, -0.446]	0.001
Contour–Contour	1.693*** (0.419)	[1.003, 2.383]	0.000
Contour–Rhythm	-0.835*** (0.117)	[-1.027, -0.643]	0.000
Contour–Note edit	-0.492*** (0.098)	[-0.653, -0.330]	0.000
Contour–Intervalic	-0.578* (0.185)	[-0.883, -0.274]	0.005
Salient leap–Salient leap	2.069*** (0.383)	[1.439, 2.700]	0.000
Salient leap–Symmetry	-1.164*** (0.309)	[-1.672, -0.656]	0.001
Salient leap–Note edit	-0.488*** (0.118)	[-0.682, -0.293]	0.000
Rhythm–Rhythm	1.949*** (0.444)	[1.219, 2.680]	0.000
Rhythm–Symmetry	-0.959* (0.323)	[-1.491, -0.427]	0.007
Symmetry–Symmetry	1.746* (0.726)	[0.552, 2.940]	0.032
Symmetry–Note edit	0.214* (0.098)	[0.054, 0.375]	0.054
Symmetry–Intervalic	0.397* (0.130)	[0.184, 0.610]	0.005
Note edit–Note edit	1.414** (0.400)	[0.757, 2.072]	0.001
Harmony–Harmony	1.916** (0.535)	[1.035, 2.796]	0.001
Harmony–Intervalic	-0.401* (0.153)	[-0.653, -0.148]	0.019
Intervalic–Intervalic	1.546*** (0.333)	[0.999, 2.093]	0.000

Middle Period: significant $\hat{\beta}$ parameters

Inference



Middle Period: significant $\hat{\alpha}$ parameters



Middle Period: significant $\hat{\beta}$ parameters

Inference

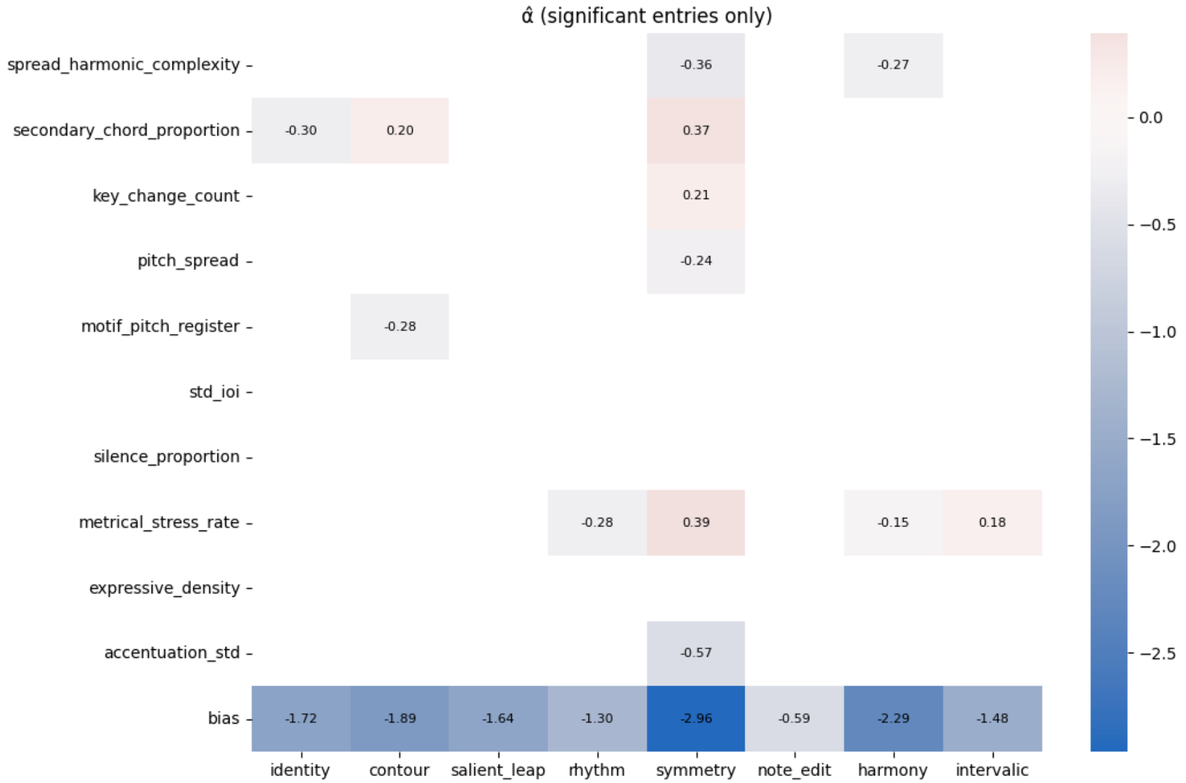
Label	Parameter	Estimate (SE)	90% CI	q^{BH}
Identity	Bias	-1.716*** (0.111)	[-1.899, -1.533]	0.000
Identity	Secondary Chord Proportion	-0.299** (0.082)	[-0.434, -0.163]	0.002
Contour	Bias	-1.889*** (0.122)	[-2.090, -1.687]	0.000
Contour	Motif Pitch Register	-0.280** (0.081)	[-0.414, -0.147]	0.004
Contour	Secondary Chord Proportion	0.201* (0.070)	[0.086, 0.317]	0.020
Salient leap	Bias	-1.645*** (0.110)	[-1.826, -1.463]	0.000
Rhythm	Bias	-1.296*** (0.169)	[-1.575, -1.017]	0.000
Rhythm	Metrical Stress Rate	-0.278** (0.083)	[-0.414, -0.141]	0.005
Symmetry	Accentuation SD	-0.572* (0.190)	[-0.885, -0.259]	0.015
Symmetry	Bias	-2.964*** (0.061)	[-3.064, -2.863]	0.000
Symmetry	Key Change Count	0.207* (0.085)	[0.067, 0.348]	0.067
Symmetry	Metrical Stress Rate	0.394*** (0.056)	[0.302, 0.487]	0.000
Symmetry	Pitch Spread	-0.242* (0.077)	[-0.369, -0.114]	0.011
Symmetry	Secondary Chord Proportion	0.370*** (0.084)	[0.232, 0.508]	0.000
Symmetry	Spread Harmonic Complexity	-0.359*** (0.066)	[-0.467, -0.251]	0.000
Note edit	Bias	-0.595*** (0.144)	[-0.831, -0.358]	0.000
Harmony	Bias	-2.288*** (0.135)	[-2.511, -2.066]	0.000
Harmony	Metrical Stress Rate	-0.147* (0.057)	[-0.241, -0.054]	0.045
Harmony	Spread Harmonic Complexity	-0.267* (0.091)	[-0.417, -0.117]	0.018
Intervalic	Bias	-1.484*** (0.171)	[-1.765, -1.203]	0.000
Intervalic	Metrical Stress Rate	0.176* (0.074)	[0.055, 0.298]	0.071

Late Period: significant $\hat{\alpha}$ parameters

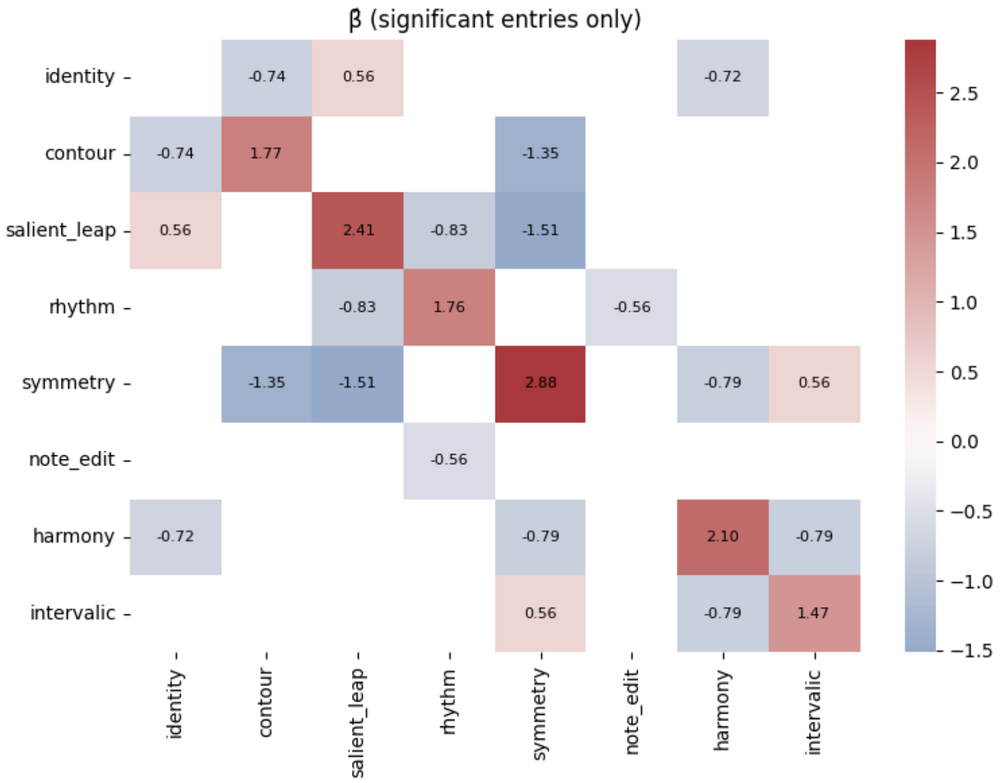
Pair	Estimate (SE)	90% CI	q^{BH}
Identity–Contour	-0.743*** (0.186)	[-1.049, -0.436]	0.001
Identity–Salient leap	0.560* (0.241)	[0.164, 0.957]	0.045
Identity–Harmony	-0.720* (0.267)	[-1.159, -0.281]	0.019
Contour–Contour	1.766* (0.660)	[0.657, 2.875]	0.025
Contour–Symmetry	-1.347* (0.509)	[-2.184, -0.511]	0.021
Salient leap–Salient leap	2.406*** (0.453)	[1.660, 3.152]	0.000
Salient leap–Rhythm	-0.827*** (0.216)	[-1.183, -0.471]	0.001
Salient leap–Symmetry	-1.512** (0.453)	[-2.257, -0.766]	0.004
Rhythm–Rhythm	1.757*** (0.405)	[1.090, 2.424]	0.000
Rhythm–Note edit	-0.563** (0.173)	[-0.848, -0.278]	0.005
Symmetry–Symmetry	2.880** (0.745)	[1.612, 4.148]	0.003
Symmetry–Harmony	-0.787* (0.280)	[-1.248, -0.327]	0.015
Symmetry–Intervalic	0.560* (0.190)	[0.247, 0.873]	0.012
Harmony–Harmony	2.098* (0.684)	[0.944, 3.252]	0.013
Harmony–Intervalic	-0.788*** (0.150)	[-1.035, -0.540]	0.000
Intervalic–Intervalic	1.465** (0.418)	[0.778, 2.152]	0.003

Late Period: significant $\hat{\beta}$ parameters

Inference



Late Period: significant $\hat{\alpha}$ parameters



Late Period: significant $\hat{\beta}$ parameters

Discussion

Discussion & Conclusions

Interpretability

- Our CRF framework parallels logistic regression but adds pairwise graph structure.
- Provides a structured, probabilistic tool for studying motif transformations.

Strengths

- Simple (relatively), interpretable parameters (α: feature effects, β: motif–motif dependencies).
- Framework not tied to a fixed segmentation: adaptable to other graph designs.
- Provides inference under sufficient effective sample size.

Limitations

- Dataset scope: only first movements, modest number of segments.
- Labels/features derived from same data, might lead to a “propagation” of the annotation errors.

Future Directions

- Larger datasets: full Beethoven corpus or multi-composer corpora.
- Medoid Motif group anchors.
- Feature-specific β matrices, mixed-effects CRFs.
- Fixed-effects controls for broader stylistic/composer comparisons.

Thanks!

Ron Taieb, Prof. Yoel Greenberg, Dr. Barak Sober