Is C Major Truly Radiant?

A Graph-Theoretic Study of J.S. Bach's Solo String Works and Their Tonal Characteristics

Introduction

- This study visualises "latent structures" in music by modelling it as a system of interacting components.

Composer \ Listener	Perceives	Does not Perceive
Intends	**Shared Structure** Musical form, counterpoint, cadences, modulation, etc.	**Obscured Structure** Structural operations such as inversion, retrograde motion, etc.
Does not Intend	**Emergent Structure** Impressions coloured by culture and experience.	**Hidden Structure** unintended, unnoticed, but measurable (e.g., 1/f fluctuation)

- "Shared structures" have long been central to music theory and music cognition.
- Our study explores how latent structures may also shape musical character.

Methods

We model pitch organisation as a network:
- Nodes = two-note transitions (2-grams)
- Edges = sequential adjacency

Dataset

We selected J.S. Bach's complete solo string works:
6 Violin Solos (BWV 1001–1006); 6 Cello Suites (BWV 1007–1012)
MIDI data: Grossman, D.J. (1997). Dave's J.S. Bach Page (http://www.jsbach.net/midi/, accessed May 2025)

BWV	Name	Key	Movements
1001	Violin Sonata No.1	G Minor	1. Adagio, 2. Fuga, 3. Siciliana, 4. Presto
1002	Violin Partita No.1	B Minor	1. Allemanda, 2. Double, 3. Corrente, 4. Double , 5. Sarabande, 6. Double, 7. Tempo di Borea, 8. Double
1003	Violin Sonata No.2	A Minor	1. Grave, 2. Fuga, 3. Andante, 4. Allegro
1004	Violin Partita No.2	D Minor	1. Allemanda, 2. Corrente, 3. Sarabanda, 4. Giga, 5. Ciaccona
1005	Violin Sonata No.3	C Major	1. Adagio, 2. Fuga, 3. Largo, 4. Allegro assai
1006	Violin Partita No.3	E Major	1. Preludio, 2. Loure, 3. Gavotte en Rondeau, 4. Menuet I, 5. Menuet II, 6. Bourrée, 7. Gigue
1007	Cello Suite No.1	G Major	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Minuet I, 6. Minuet II, 7. Gigue
1008	Cello Suite No.2	D Minor	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Minuet I, 6. Minuet II, 7. Gigue
1009	Cello Suite No.3	C Major	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Bourrée I, 6. Bourrée II, 7. Gigue
1010	Cello Suite No.4	E ♭ Major	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Bourrée I, 6. Bourrée II, 7. Gigue
1011	Cello Suite No.5	C Minor	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Gavotte I, 6. Gavotte II, 7. Gigue
1012	Cello Suite No.6	D Major	1. Prélude, 2. Allemande, 3. Courante, 4. Sarabande, 5. Gavotte I, 6. Gavotte II, 7. Gigue

Why These Works?
- Homotonal design: all movements in the same key → consistent analysis
- For strings: avoids temperament/tuning issues(*)
- Unaccompanied texture: clarifies internal pitch relations
- Resonance sensitivity: reflects J.S.Bach's tonal colour & structural balance

(*) Although analysis uses MIDI (independent of temperament), historical tuning systems may still have influenced Bach's key choices and tonal design.

Defining "Radiance"

To quantitatively explore the idea of "radiance" in music, we define a Radiance Score $R(x_1, x_2, x_3, x_4)$ as a weighted linear combination:

$$R(x_1, x_2, x_3, x_4) = ax_1 + bx_2 + cx_3 + dx_4$$

where
- x_1 =1 if Major, 0 if Minor
- x_2 =1 if Tonic matches an open string, 0 otherwise
- x_3 =1 if Tonic matches a lower open strings, 0 otherwise
- x_4 =1 if Dance-based, 0 if Sonata/Fuga

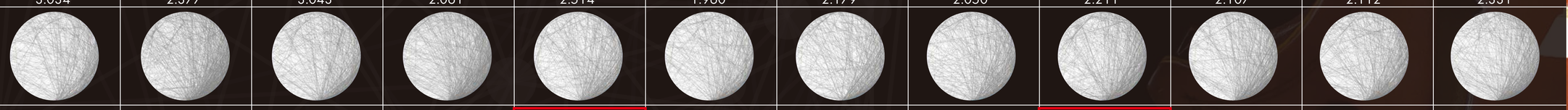
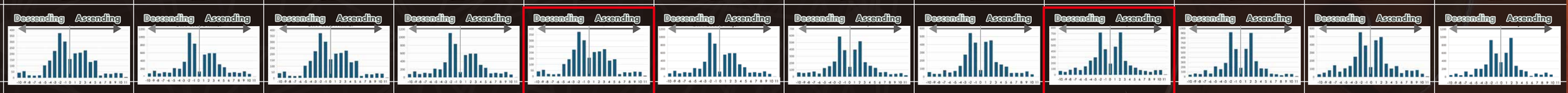
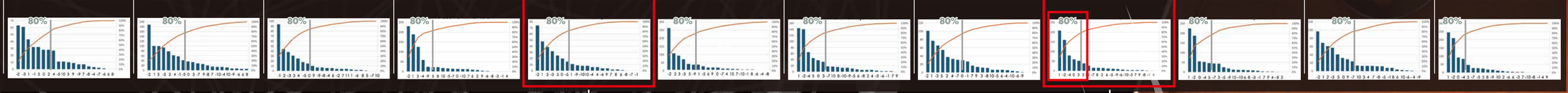
The weights were heuristically determined as follows:

Weight : a=2, b=1, c=1, d=2

BWV	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012
Key	G Minor	B Minor	A Minor	D Minor	C Major	E Major	G Major	D Minor	C Major	E♭Major	CMinor	D Major
x_1	0	0	0	0	1	1	1	0	1	1	0	1
x_2	1	0	1	1	0	1	1	1	1	0	1	1
x_3	1	0	0	1	0	0	1	0	1	0	1	0
x_4	0	1	0	1	0	1	1	1	1	1	1	1
Radiance Score	2	2	1	4	2	5	6	3	6	4	4	5

(Hevner 1935; Crowder 1985; Hunter et al. 2008; Young 1991; Fletcher & Rossing 1998; Williams 2003; Stowell 2001)

Results

Radiance Score	2	2	1	4	2	5	6	3	6	4	4	5
BWV	**1001** G Minor	**1002** B Minor	**1003** A Minor	**1004** D Minor	**1005** C Major	**1006** E Major	**1007** G Major	**1008** D Minor	**1009** C Major	**1010** E♭ Major	**1011** C Minor	**1012** D Major
Number of Nodes	344	435	348	452	327	331	311	320	360	389	345	441
Number of Edges	2564	6262	2997	5725	3176	3159	3639	3389	4544	5152	3645	5486
Avg. Number of Neighbors	5.424	6.060	5.569	6.394	5.284	5.583	5.498	5.688	5.933	5.681	5.791	5.800
Network Diameters	9	8	8	9	11	8	9	9	8	8	8	8
Network Radius	5	5	5	5	6	5	5	5	5	5	5	5
Characteristic Path Length	3.916	3.675	3.799	3.716	4.263	3.739	3.669	3.675	3.633	3.746	3.626	3.886
Clustering Coefficient	0.048	0.029	0.045	0.039	0.072	0.051	0.051	0.049	0.038	0.046	0.053	0.080
Network Density	0.016	0.014	0.016	0.014	0.016	0.018	0.018	0.018	0.017	0.015	0.017	0.013
Network Heterogeneity	**0.762**	**0.826**	**0.790**	**0.824**	**0.747**	**0.837**	**0.837**	**0.796**	**0.849**	**0.824**	**0.817**	**0.869**
Network Centralization	0.043	0.046	0.051	0.055	0.049	0.063	0.063	0.061	0.053	0.053	0.059	0.046
Power Law Fitting	Yes	Yes	Yes	Yes	Yes	Log Normal may better fit	Yes	Yes	Yes	Yes	Yes	Yes
Power Law Exponent	3.034	2.377	3.043	2.061	2.514	1.960	2.179	2.050	2.211	2.107	2.112	2.331
Network Visualisation												
Interval Frequency Analysis												
Pareto Analysis												

Symmetry between Ascending and Descending Intervals

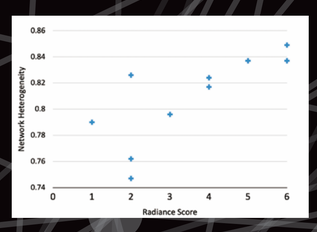
Higher Concentration toward Tonic

Graph-Theoretic Analysis

Most of Bach's solo string works show **scale-free network structures**, with node degrees following a power-law distribution. → Suggests **self-organising pitch-transition systems** (Barabási & Albert, 1999).
- Degree distributions were assessed via maximum likelihood estimation (MLE) and compared to alternative models (exponential, log-normal).
- All but one work (BWV 1006) followed a power-law; BWV 1006 fit slightly better to a log-normal distribution.

Radiance Score vs Network Heterogeneity

Network heterogeneity quantifies the variability in node degree across the network, and showed a meaningful correlation with the Radiance Score (Spearman's ρ = 0.82, p = 0.001).



Underlying Structural Features

To explain the correlation between Radiance Score and network heterogeneity, we conducted:
(1) Interval Frequency Analysis – frequency of each pitch interval (in semitones) across 2-grams
(2) Pareto Analysis – concentration of transitions toward the tonic

These revealed three key traits in high-Radiance works:
- **Symmetry between ascending and descending intervals** (e.g., +4/–4)
- **Lower Pitch Interval Variety** (fewer interval types overall)
- **Greater tonic focus** (few 2-grams dominate tonic motion)

→ Suggests: Radiance may emerge from **simpler, focused transition networks**.

Structural Contrast Between Two C Major Works: BWV 1005 vs. 1009

Despite sharing C major, BWV 1009 (Cello Suite No.3, Radiance Score 6) and BWV 1005 (Violin Sonata No.3, Radiance Score 2) differ structurally.
BWV 1009: Greater interval symmetry and concentrated transitions to the tonic → higher Radiance Score & network heterogeneity.
BWV 1005: Greater interval diversity and distributed tonic approach → lower Radiance Score.

Results_Summary

Radiant pieces showed higher network heterogeneity, with fewer pitch transitions occurring more frequently.
C major did not necessarily imply greater radiance.
Structural simplicity and regularity may be contributing factors.

Conclusion

- In Bach's solo string works, radiant character aligned with latent structural features - symmetry, simplicity, and tonal focus.
- Future work will apply this approach to full key-cycle works to examine how radiance related structures vary across tonalities and compositional styles.

About Us

Miya Kohno
Doctoral Student, Graduate School of System Design and Management, Keio University, Yokohama, Japan

Yo Tomita
Professor of Musicology, Department of Music, School of Arts, English and Languages, Queen's University Belfast

Masahiro Niitsuma
Associate Professor of Graduate School of System Design and Management, Keio University, Yokohama, Japan

Codes and Data (Github)

Presentation