From Theory to Topology: Modeling Turkish Makam Complexity with Multiplex Networks and Deep Learning

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Outline

Background and Related Work

- Previous Research on Modeling Turkish Makam Music
- MakamNetz
- Theory of Relations

Previous Research on Modeling Turkish Makam Music

- Modeling Turkish Makam Music proposed several challenges in computational modeling.
- Past computational methods mostly relied on 20th century theoretical models that mapped *perde* (pitch) and *çeșni* (tetrachordal and pentachordal sub-units to form melodies) relations over a Euclidean space (Bozkurt et al., 2014; Atlı et al., 2014; Karaosmanoğlu, 2012).
- Those spaces divide a whole tone into 9 equal units (*koma*) and create seemingly transposable units in a closed space. Nevertheless, most of the space is unused in practice, thus rendering it partial.
- This type of modeling often causes discrepancies between theory and practice, thus challenging computational models as well (Bozkurt et al., 2014).

Inspirations for **MakamNetz**

- Hızır Ağa, *Tefhîmü'l Makâmât fî Tevlîdi'n Nagâmât* — among the first graph-like models of *makamlar*.
- Dmitri Tymoczko, *A Geometry of Music* (2011).
- Johnny Farraj & Sami Abu Shumays, *Inside Arabic Music* (2019).
- Ozan Baysal, *Zaman Makam Analizi ve Ferahfeza Makam Seyri içerisinde yer alan Perde–Çeşni ilişki, hareket ve dönüşümleri* (2018).

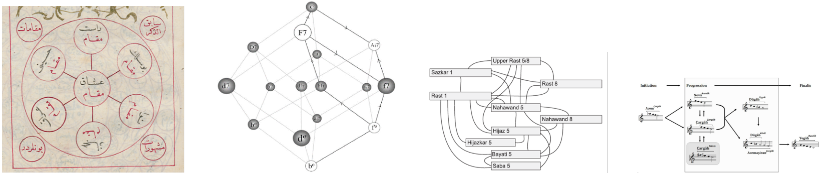
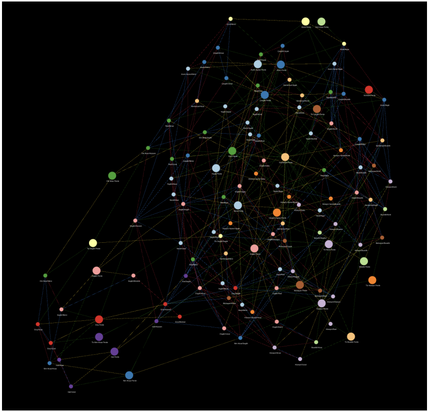


Figure: Figures from works that inspired MakamNetz

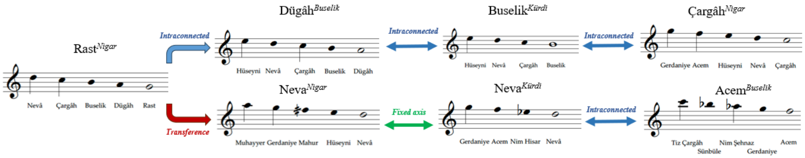
MakamNetz and Mapping the Makam Universe

- **MakamNetz** (Baysal et al., 2024) has been proposed as a topological model to capture relationships between different melodic subunits in the Turkish Makam Universe.
- It enables the study of **melodic paths, çeşni relations, graph subspaces, graph motives**, and **topology**.
- Explore the MakamNetz Frontend: https://makam-netzfrontend.vercel.app/

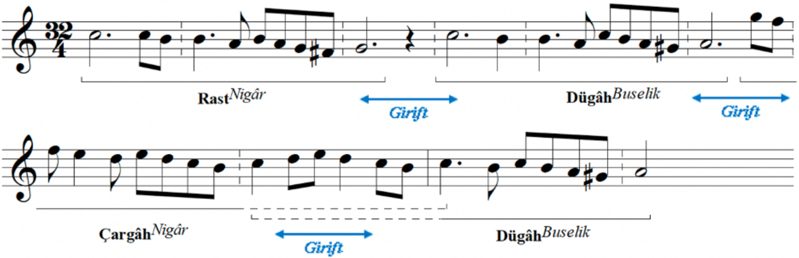


Construction of the Map

- The global map has been built based on progression descriptions from historical theoretical sources from 15th till 20th century as well as repertoire analysis and practical knowledge
- Each node corresponds to a melodic unit. (Perde$^{\text{Çeşni}}$)
- Each edge corresponds to one of the 4 types of relations

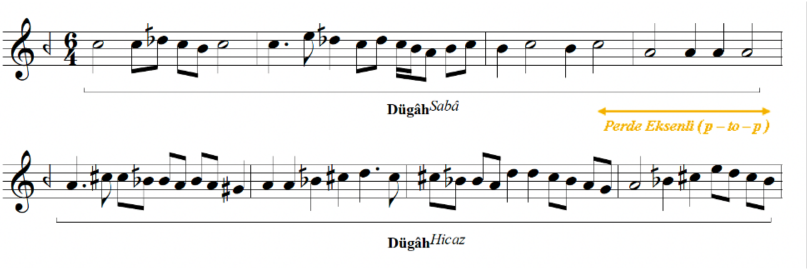


Relations of Auditory Proximity: **Girift (intraconnected)**

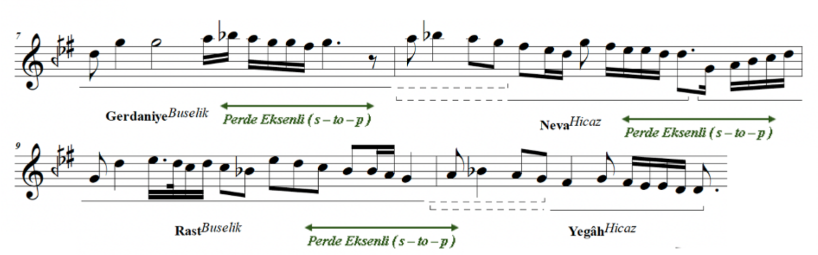


Relations of Harmonic Similarity: **Transference (t-related)**

Relations of Modal Shift: **Principal Axis Relation (p-related)**



Relations of Scale Construction: Secondary Axis ↔ Principal Axis Connection **(s-to-p or p-to-s)**



Motivation

- Prior graph models in musicological work define topology but generally do not propose analytical methodology for analysis. They serve merely as maps and are used to validate existing theoretical or practical grammars.
- Graphs are generally **unweighted** and **undirectional**, hence do not propose a framework sufficient for building progressions, trajectories, or understanding the **effects of different types of relationships** on perde/çeşni progressions.
- In most cases, calculating compositional choice mechanisms for graph walks in an explainable fashion can be practically **intractable** due to the extensive number of options.
- The **effects of topology** and **different types of relations** in the graph have been unexplored in the context of Turkish Makam Music.

Contribution and Aim of the Work

- Propose a **computational method** for studying the MakamNetz universe in order to understand the workings of Turkish Makam Music, based on **deep learning methods** and **network science**.
- Explore the effects of topology, auditory relationships, and theoretical constructions in practice.
- Explore the **effects of latent space representation** on the ways in which subspaces and progressions are constructed.
- Propose and evaluate a **flexible computational model** that can be used to test and study convergence and divergence between theoretical descriptions and repertoire, as well as to explore progression mechanics and subspace construction.

Data

- 3-minute audio recordings of improvisations of 11 *Perde–Çeșni* classes from 4 different instruments (two types of *ney*, *lavta*, and clarinet), approximately 120 minutes in total — divided into 8-second clips (50% overlap) and grouped as train, validation, and test sets.
- 154 *Makam* descriptions from historical sources transformed into the MakamNetz graph model (4–6 nodes each).
- 20-measure *Perde–Çeșni* sequences from the analysis of the repertoire (62 pieces) (*Yalçın, 2024; Kesgin, 2025; MakamNetz project*).

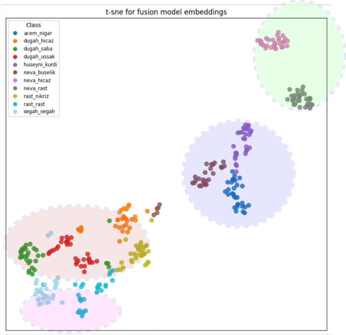
Methodology

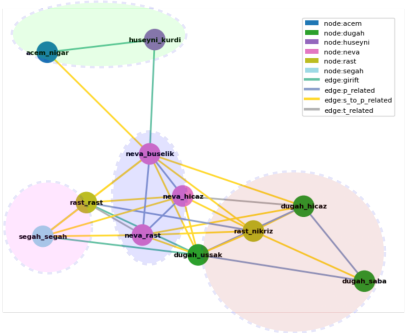
- Training a deep classifier and constructing the latent space
- Modeling progression mechanics sampling from latent space with Monte Carlo Markov Chain (MCMC) with RBF kernel
- Testing progression parameters according to network science metrics and tuning temperature (τ) and kernel width (σ)
- Sampling methods for evaluation

Training a Classifier

Model	Accuracy (Test)
Baseline Logistic Regression	0.77
Baseline MERT	0.73
Conformer	0.80
Bi-LSTM + Self-Attention	0.91
CNN-LSTM Fusion	**0.97**

Comparison of Latent Representation of *Perde–Çeşni* Classes and Their Topology





Left: Two-dimensional t-SNE visualization of latent representations extracted from the penultimate layer of the classifier after applying PCA and t-SNE.
Right: Topological map of the 11 *Perde–Çeşni* classes used in the experiments. Corresponding regions are shaded with the same colors to illustrate how the learned latent space aligns with the theoretical topology of the MakamNetz model.

Sequence Simulation: Monte Carlo Markov Chains

- Sample a representative z_i for class k from the latent space with probability according to distance from class mean

$$w_{ik} \propto \exp\left(-\frac{\|z_i - \mu_k\|^2}{2\tau^2}\right)$$

- With representatives $z_1, \ldots, z_k$ generate a Markov chain with transition probabilities based on embedding distances

$$P_{ij}^{L} \propto \exp\left(-\frac{\|z_i - z_j\|^2}{2\sigma^2}\right)$$

Sequence Simulation: Monte Carlo Markov Chains

- Apply a theory mask $G \in \{0,1\}^{K \times K}$ to only allow MakamNetz transitions

$$P_{ij}^{LT} \propto P_{ij}^{L} \odot G$$

- τ and σ control the temperature of within class sampling and between class transitions, respectively
- Theory Only-Model: $P_{ij}^{T} \propto G_{ij}$:
- Average over MC runs: $\bar{P}(\tau, \sigma)$

Exploring Progression Behavior: Tuning τ and σ

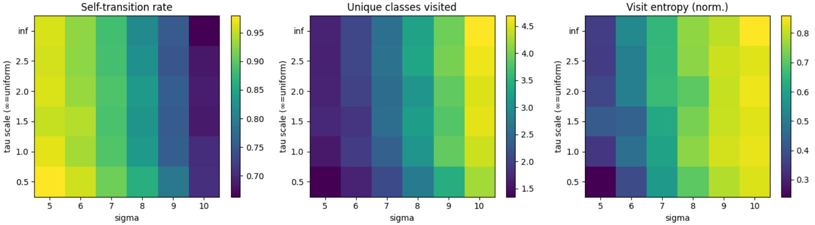


Illustration of the effect of tuning the progression parameters τ (transition threshold) and σ (relation weighting) on the model's trajectory generation. Lower τ values yield denser transition networks (parsimoniously explore), while higher σ sharpens relation selectivity. (transition to distant samples)

Exploring Progression Behavior: Evaluating Model Simulations

- Simulations conducted to assess the model's ability to reproduce dynamic graph properties observed in repertoire sequences.
- **Evaluation Target:** *20-measure sequences from the repertoire.*
- **Compared Models:**
  - Random walk on Erdős–Rényi (null model)
  - Random walk on Havel–Hakimi (degree-preserving)
  - Random walk on MakamNetz graph
  - Proposed model simulations
- **Evaluation Metrics:**
  - Average node frequency
  - Mean edge weight
  - Number of directed cycles
  - Graph density
  - Node frequency entropy

Exploring Space Dynamics: Sampling from Sequences Across Datasets

- Performed 1000 sampling runs from three sources: repertoire (R), makam descriptions (D), and model simulations (S).
- For each run i, random 20-measure sequences were drawn from R_i, bootstrapped subsets were sampled from D_i, and simulated sequences were generated from S_i.
- Each sampling yields an edge set E_i used to construct a weighted adjacency matrix:

$$W_i(u, v) = \sum_{(u,v) \in E_i} 1$$

- Mean and standard deviation across runs:

$$\mu(u, v) = \frac{1}{N} \sum_i W_i(u, v), \quad \sigma(u, v) = \sqrt{\frac{1}{N-1} \sum_i (W_i(u, v) - \mu(u, v))^2}$$

Exploring Space Dynamics: Standardization and Divergence Analysis

- Standardized edge weights for comparison:

$$Z(u,v) = \frac{W(u,v) - \mu(u,v)}{\sigma(u,v)}$$

- Jensen–Shannon divergence computed pairwise between distributions from repertoire, descriptions, and simulations:

$$JS(P_A \parallel P_B) = \frac{1}{2}KL(P_A \parallel M) + \frac{1}{2}KL(P_B \parallel M), \quad M = \frac{1}{2}(P_A + P_B)$$

- This quantifies structural and probabilistic similarity across theoretical, empirical, and simulated network spaces.

Exploring Space Dynamics: Distribution of Edge Occurences

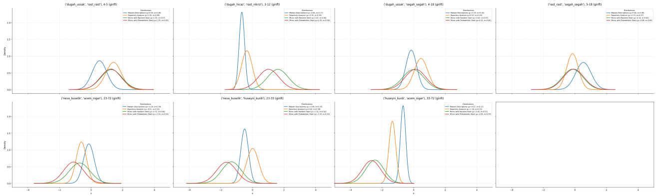


Figure: Edge occurrence probability distributions across datasets and models grouped by relation

This graph shows only edges with relation attribute: girift (intraconnected). Model performance show no significant difference between random starts vs. starting node according to frequency distribution of nodes across makam definitions.

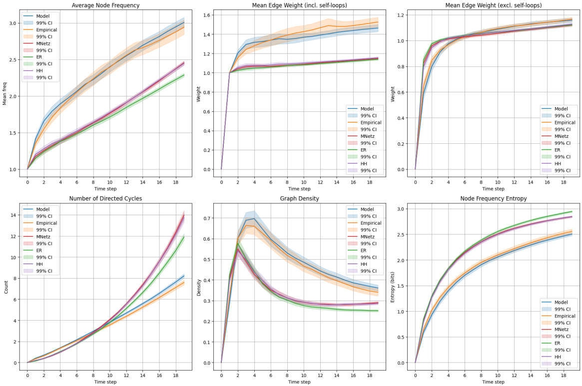
Exploring Transition Dynamics

- **Training Target:** Makam Description Matrix P^{D}_{ij}
- Fit Model Parameters

$$(\hat{\tau}, \hat{\sigma}) = \arg \min_{\tau,\sigma>0} \frac{1}{K} \sum_{i=1}^{K} \mathrm{KL}\left(P^{D}_{i\cdot} \,||\, \bar{P}_{i\cdot}(\tau, \sigma) \right)$$

- Null-Models: Geometry Medians for τ, σ
- **Evaluation Target:** Corpus Transition Matrix P^{C}_{ij}
- **Metrics:**
  - Mean Row Spearman ρ: Ranking Alignment
  - Mean Row *JS*-Divergence: Probability Similarity
  - Prediction Accuracy

Results: Model Performance on Exploring the Perde/Çeșni Space and Temporal Dynamics of Sequences



Comparison of space exploration

Network dynamics of a 20 measure *çeșni* progression across null models, empirical data and sequences created by the model (*tau*: 15, *sigma*: 4)

Results: JS Divergence Comparison

Relation Type	JS(descriptions–repertoire)	JS(mcmc–descriptions)	JS(mcmc–repertoire)
girift	0.423506	0.527596	**0.404156**
p_related	0.245721	0.596760	**0.482004**
s_to_p_related	0.352542	**0.435572**	0.436567
t_related	0.436363	0.220138	**0.210343**
Overall Mean	0.357367	0.473322	**0.424110**

Table: JS divergence between makam descriptions, repertoire analysis, and model progressions.

Lower is closer. Overall model behavior resembles repertoire practice which relies more on auditory relations than theoretical constructions. For relations that rely on aural proximity (girift, p_related), the model favors repertoire practices, whereas for scale-building relations (t_related, s_to_p_related) no significant difference has been observed..

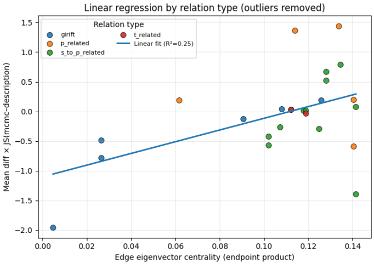
Results: Transition probabilities and Prediction Accuracy

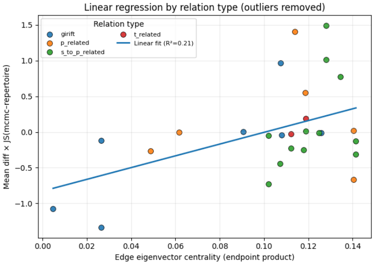
Family	**Variant**	Spearman ρ	JS-Divergence	Top-1
T	Fixed	**0.6565**	0.1423	0.1705
L	Null	0.3423	0.1818	0.1474
L	Fitted	0.3440	0.1896	0.1740
LT	Null	0.5568	0.1421	**0.2275**
LT	Fitted	0.5641	0.1422	**0.2275**

Table: Comparison of transition models combining topology (T) and latent space (L).

The topology-only model (T) preserves theoretical transition rankings, while the latent-only model (L) captures stylistic variability but diverges from theory. Their combination (LT) achieves the highest predictive accuracy, showing that musical progression emerges from the interplay between **structural topology** and **learned stylistic dynamics**. “Null” and “Fitted” refer to whether the parameters τ and σ were tuned or kept fixed.

Effect of Topology on Model Behavior





Relationship between eigenvector centrality and JS divergence.

As node importance increases, the model tends to overestimate edge probabilities, suggesting that topology begins to dominate and distort latent space effects. Conversely, less central nodes are underestimated—an outcome of hard masking via the adjacency matrix.

From Results to Interpretation

We now move from model validation to musical interpretation.

How does topology actually shape stylistic behavior in Makam music?

The following discussion reframes the results
as an explanation of how **network structure governs melodic logic**.

Discussion – The Musical Logic of Topology (I)

- **Topology in MakamNetz** is not just structural—it **governs musical motion** and shapes stylistic choices.
- The **latent space** captures *stylistic proximity* between perde–çeșni classes, while topology defines the *grammar of movement*.
- In sequence simulations, **central nodes** exert stronger attraction—
  leading to **overrepresentation of familiar regions**.
- **Peripheral zones** remain underexplored, reflecting how performances often return to tonal centers rather than distant modulations.

Discussion – The Musical Logic of Topology (II)

- **Lower divergence between model and repertoire** shows that practice internalizes topological constraints more faithfully than theory.
- Topology acts as a **cognitive and stylistic attractor**, balancing:
  - τ: coherence and local similarity
  - σ: exploration and modal shift
- This interaction explains how **makam progressions achieve both stability and expressive fluidity**.
- Together, **topological structure + learned representation** offer a new, generative explanation of **how musical grammar emerges from networked relations**.

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Explore our interactive tools and watch demonstrations of the MakamNetz project.

Thank You!

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