Entropy-Based Critical Points in Overtone Interaction Predict Cross-Cultural Pitch Stabilization in Maqam and Raga Performance

Oral Presentation
ICCCM 2025

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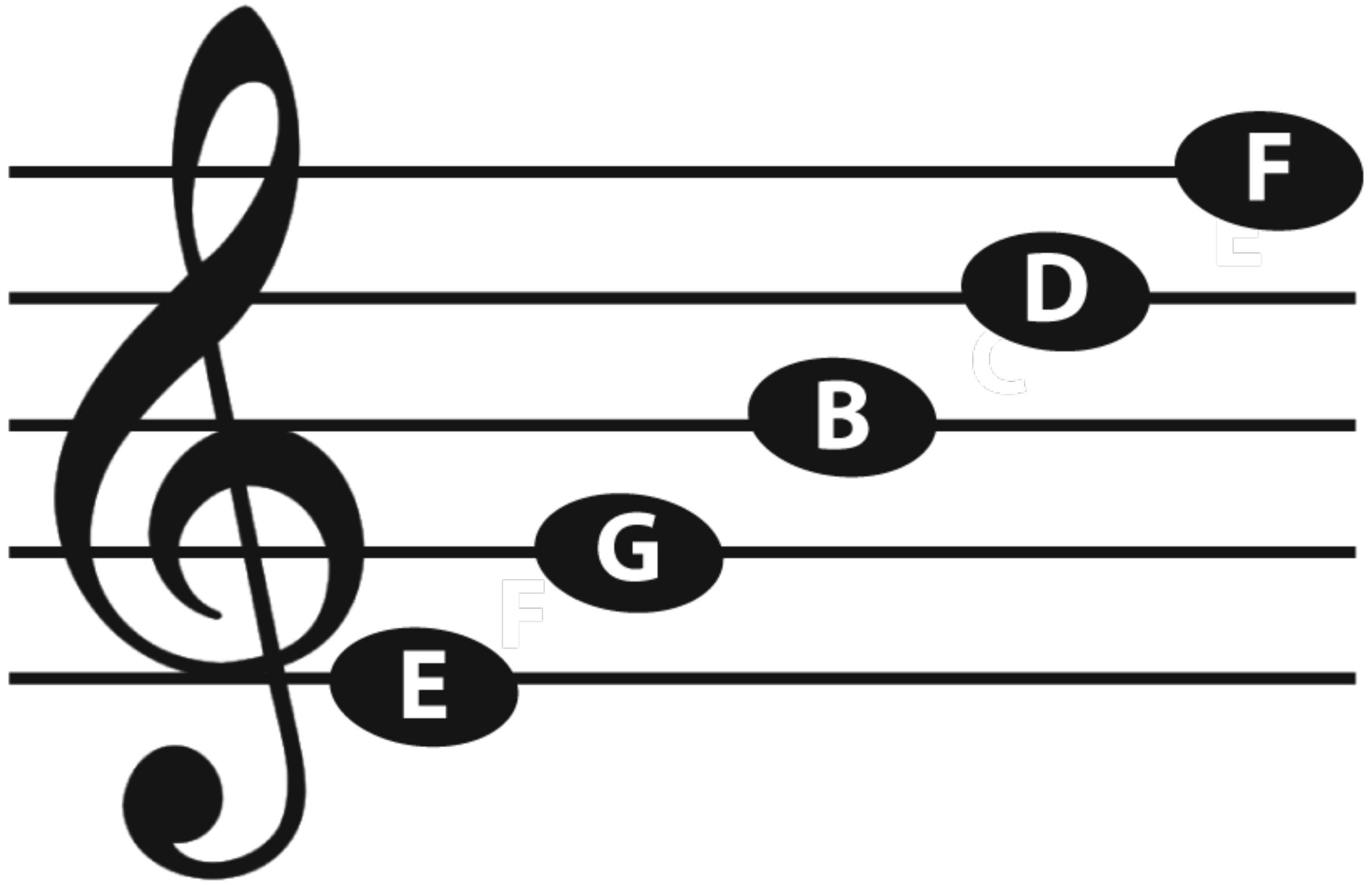
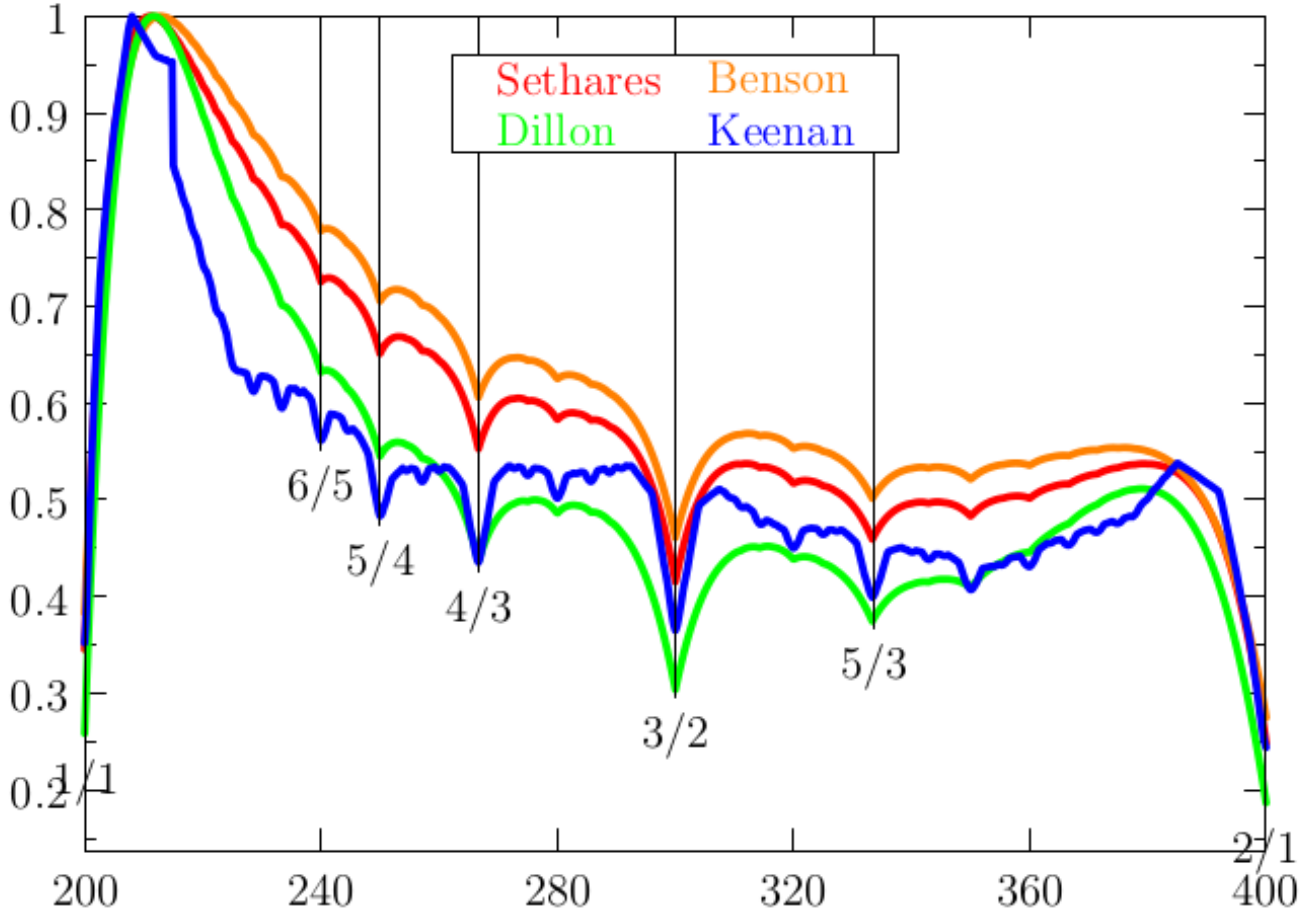
Why do musical cultures stabilize on particular pitches?



Roughness models

Helmholtz -> Plomp & Levelt -> Sethares

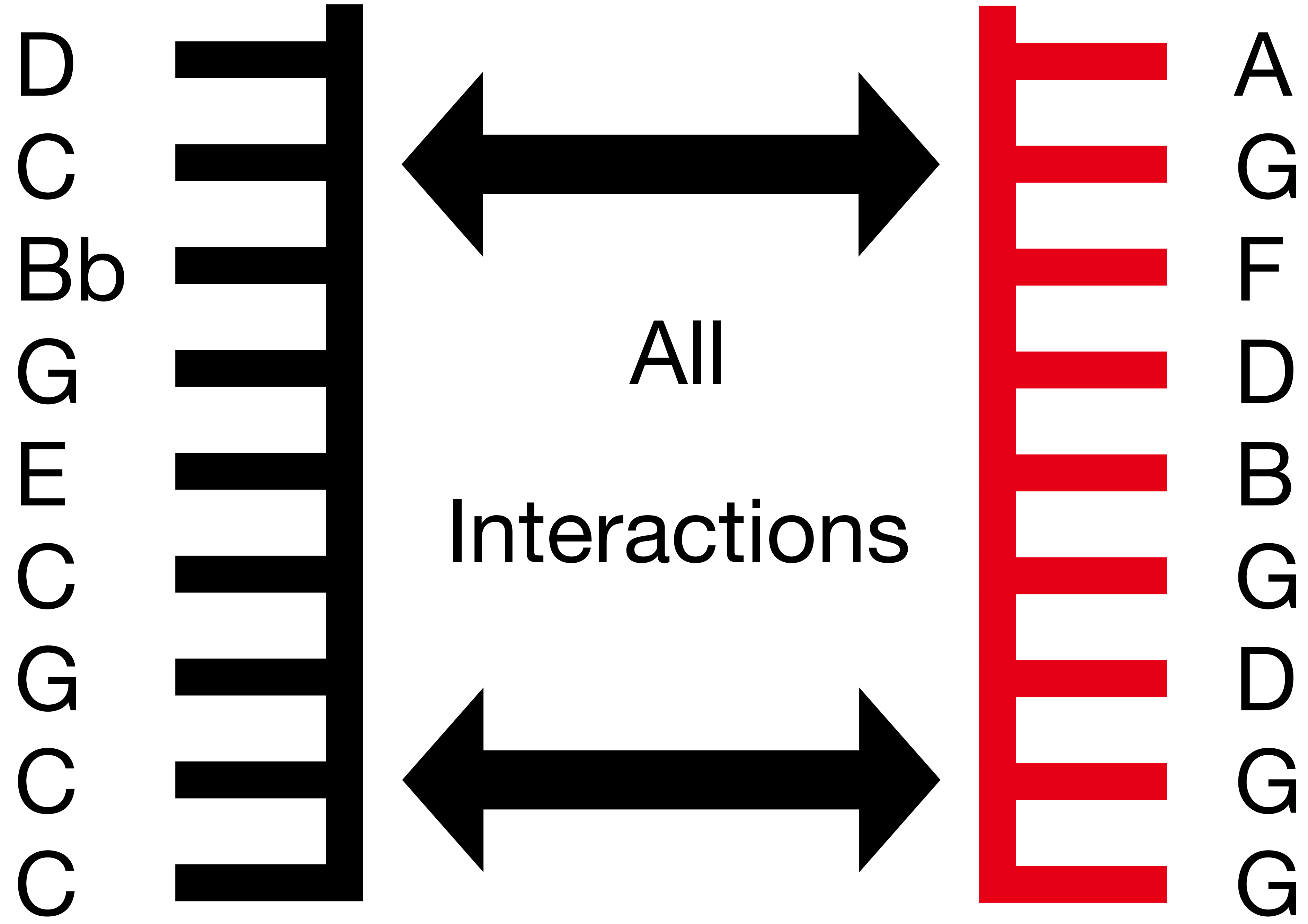


Sethares (1993, 2005), Benson (2007), Dillon (2017), Keenan (2019)

New Representation

Overtone Interaction Vector

Sethares $$D = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{ij} \, e^{-k\,|f_{1i} - f_{2j}|}$$



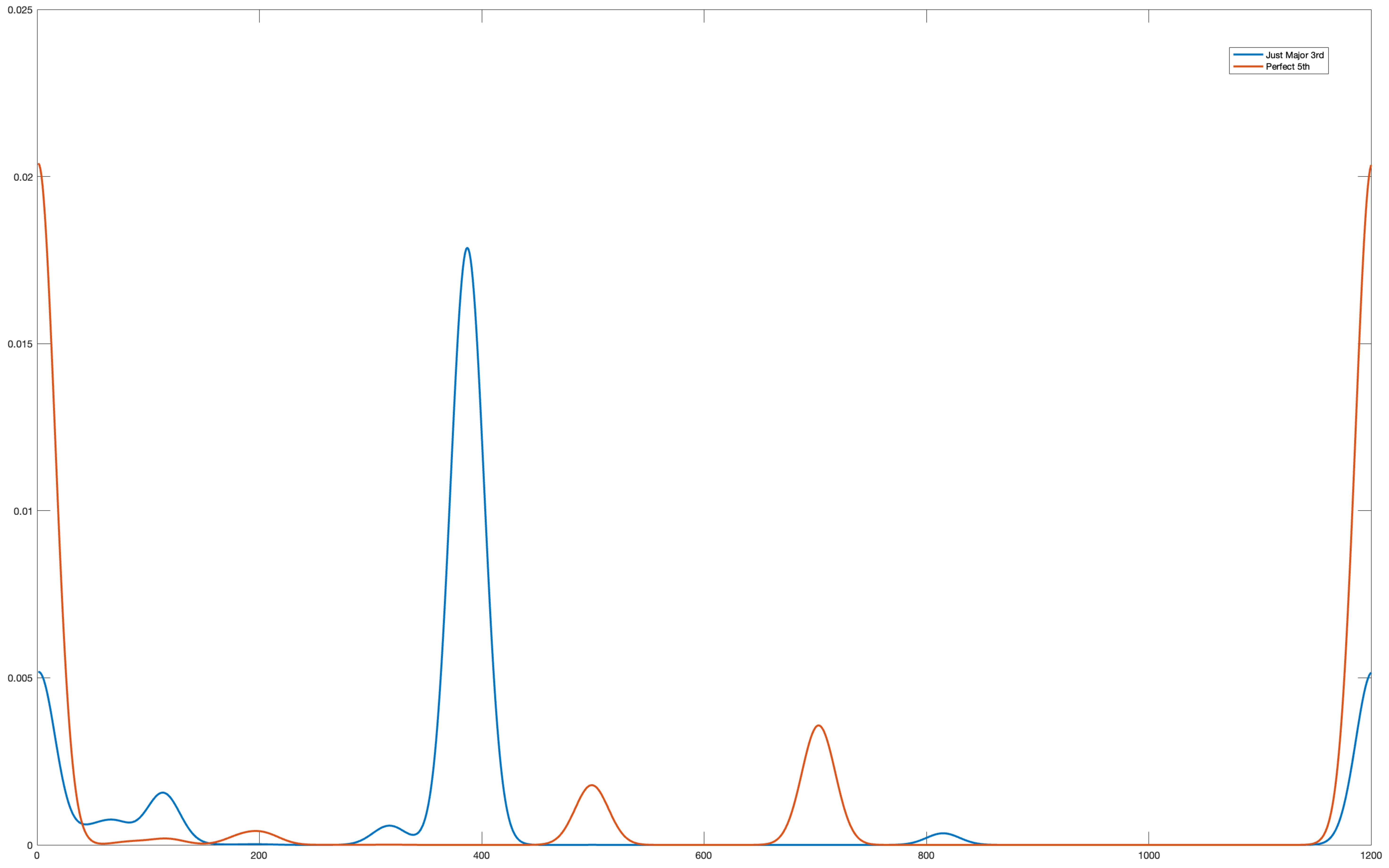
$$V(x) = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{ij} \exp\big(- m_k \, |f_1 i - f_2 j| \big) \exp\left(- \frac{1}{2} \left(\frac{x - \mu_{ij}}{\sigma} \right)^2 \right)$$

$$\sigma = \frac{\mathrm{JND}_{\mathrm{cents}}}{1200} \cdot G$$

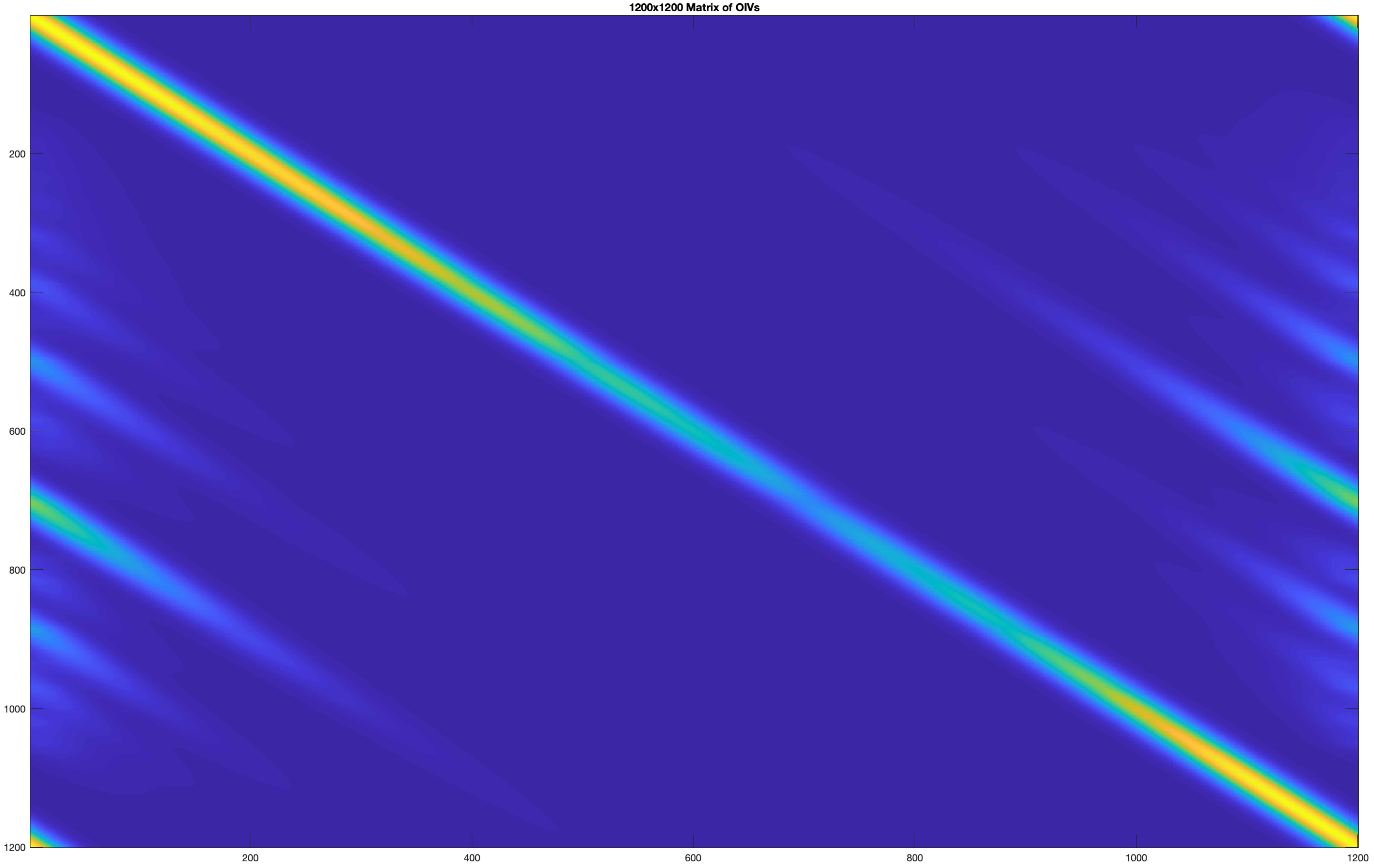
$$\mu_{ij} = \left| G \cdot \log_2 \left(\frac{f_2 j}{f_1 i} \right) \right| \pmod G$$

Overtone Interaction Vector

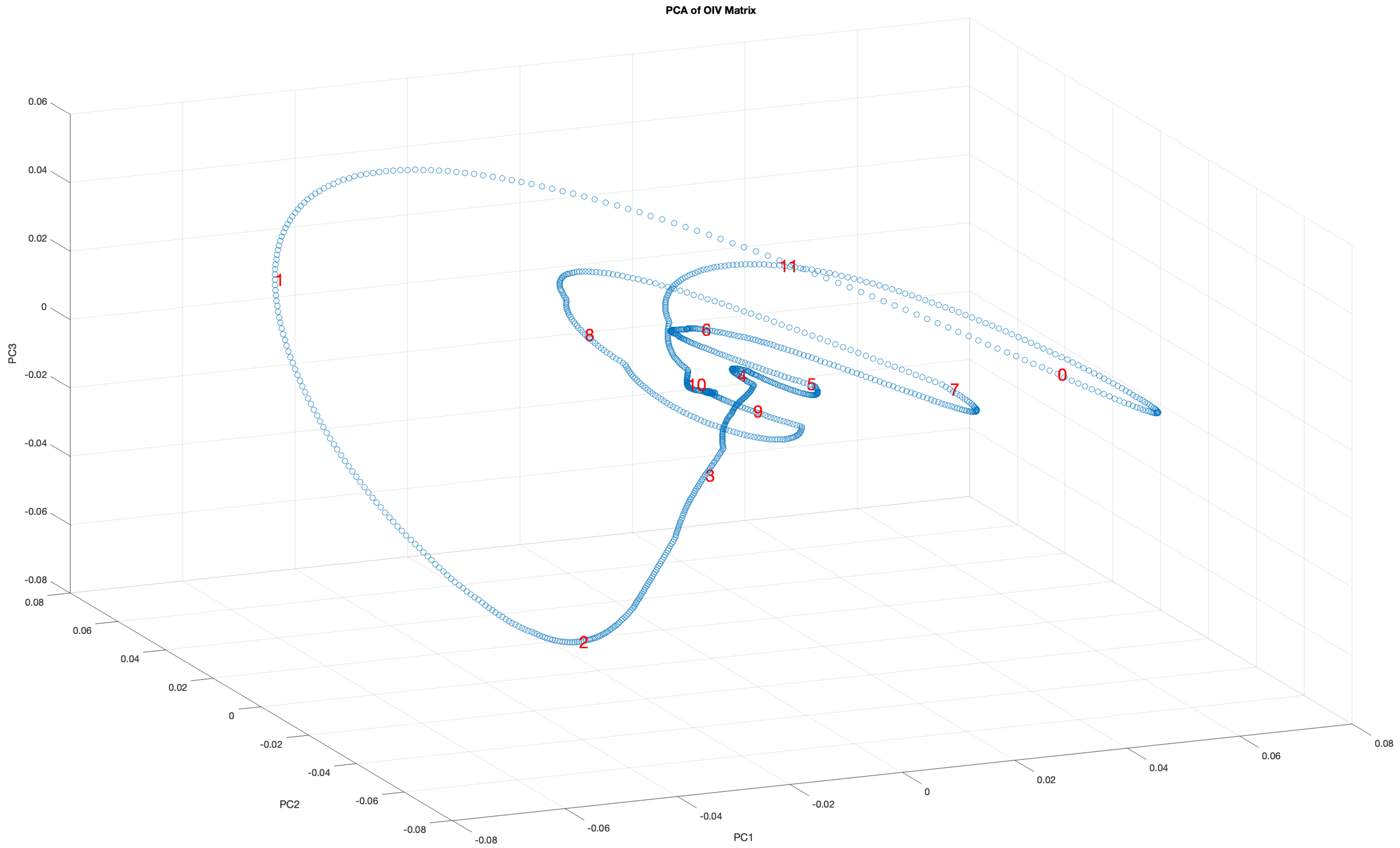
Example: Just Major 3rd, Perfect 5th



Matrix of OIVs

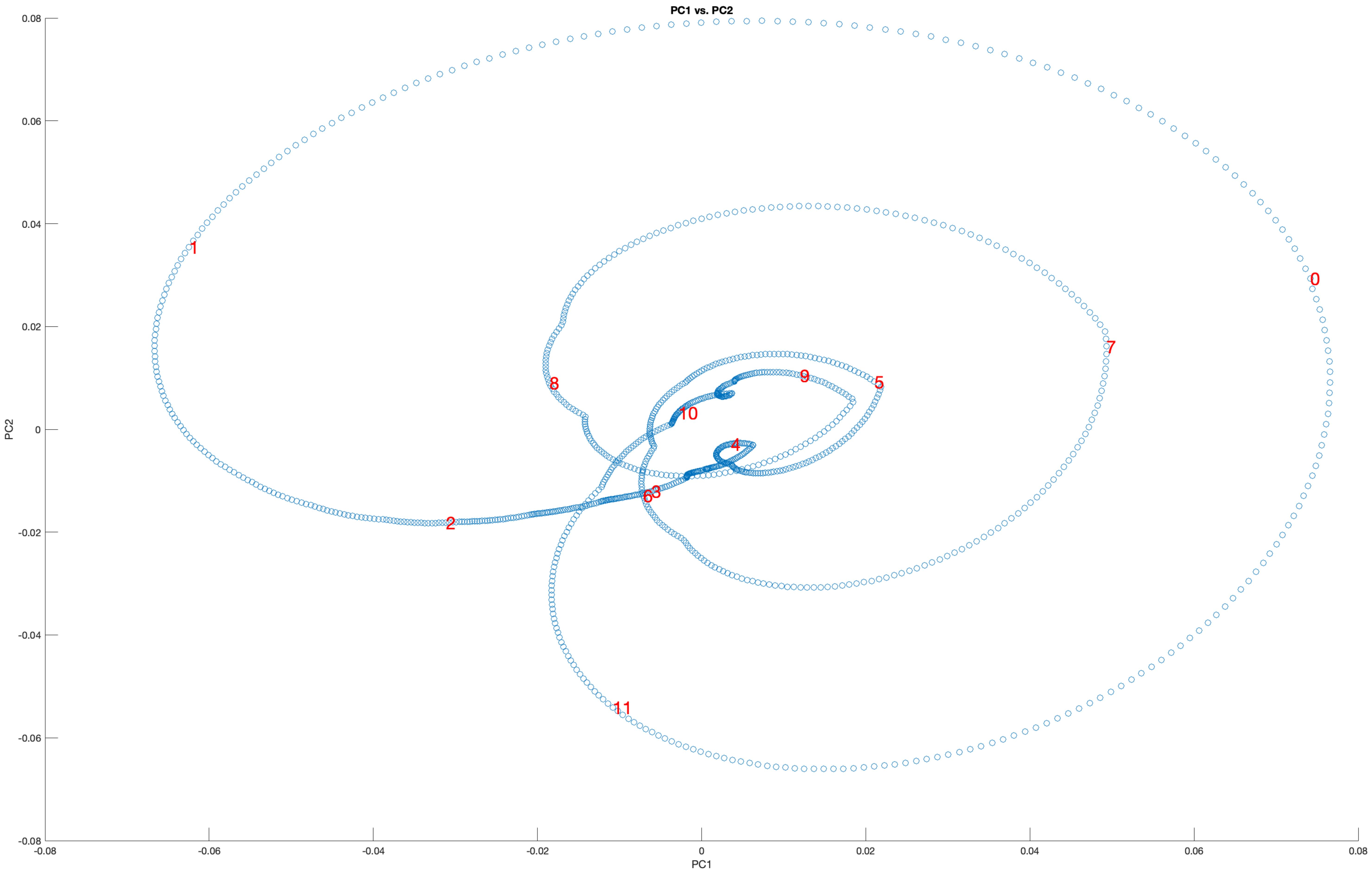


PCA decomposition of OIV matrix



PC1 correlates with dissonance

r = 0.9, p < 0.0001 (with data from Masina et al. 2022)



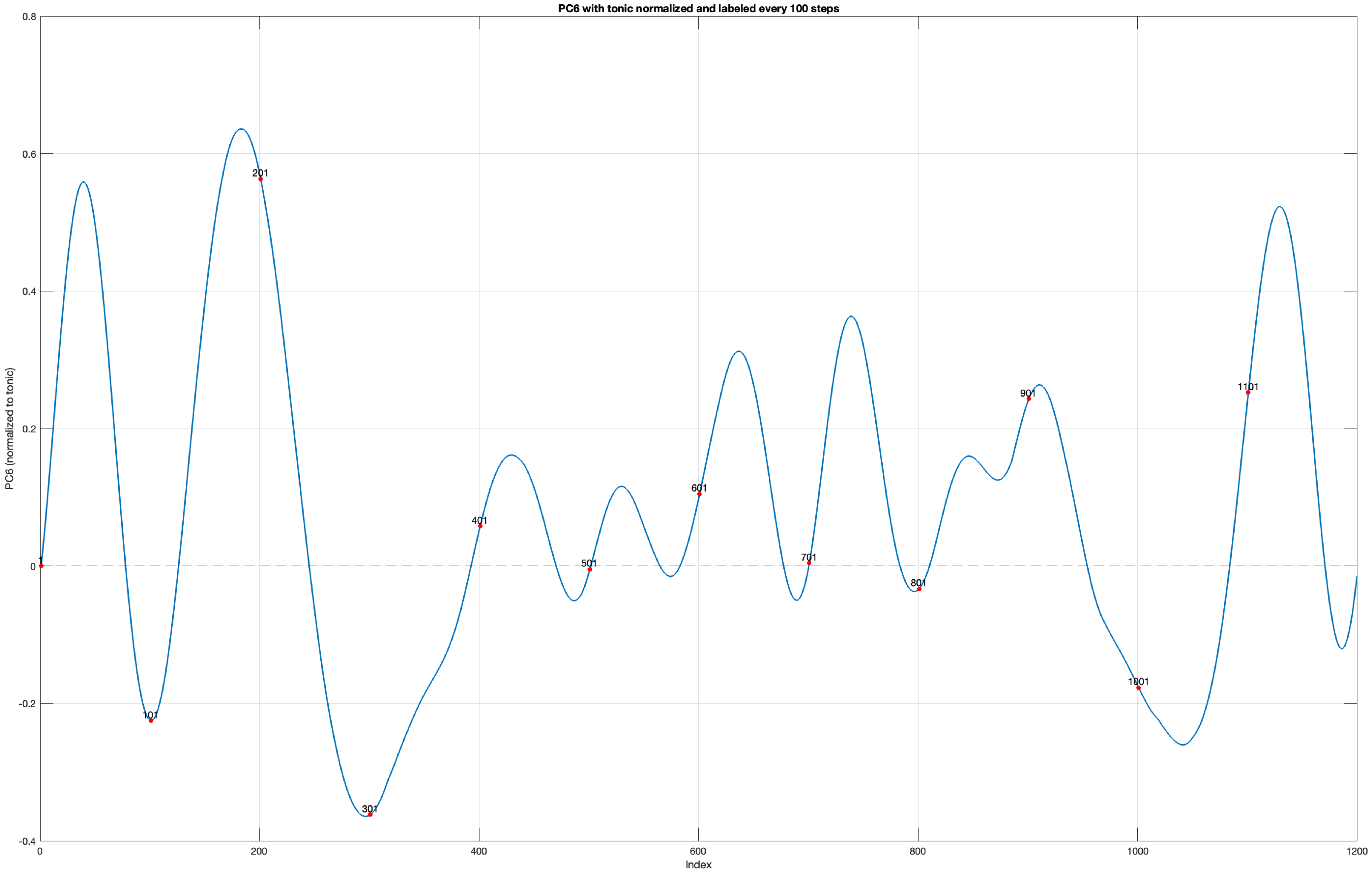
What about PC2, PC3, PC4, etc.?

PC1 explains ~8.5% of variance

- PC2 = 8.2645%
- PC3 = 6.8271%
- PC4 = 6.5668%
- PC5 = 5.6734%

- What other features are we responding to?
- Are there more dimensions to these sounds?

PC6: Major vs. minor interval quality?

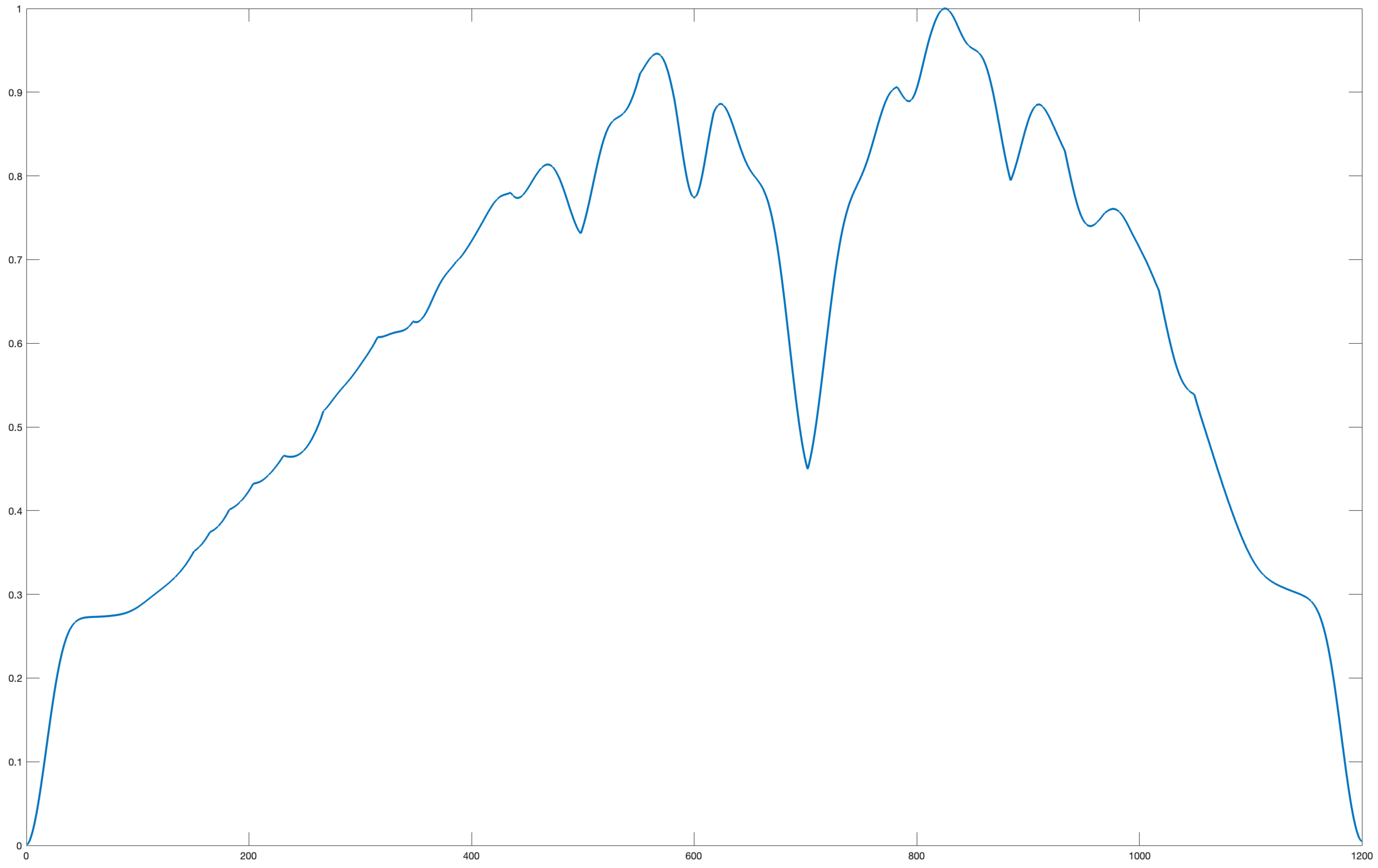


Summary of model

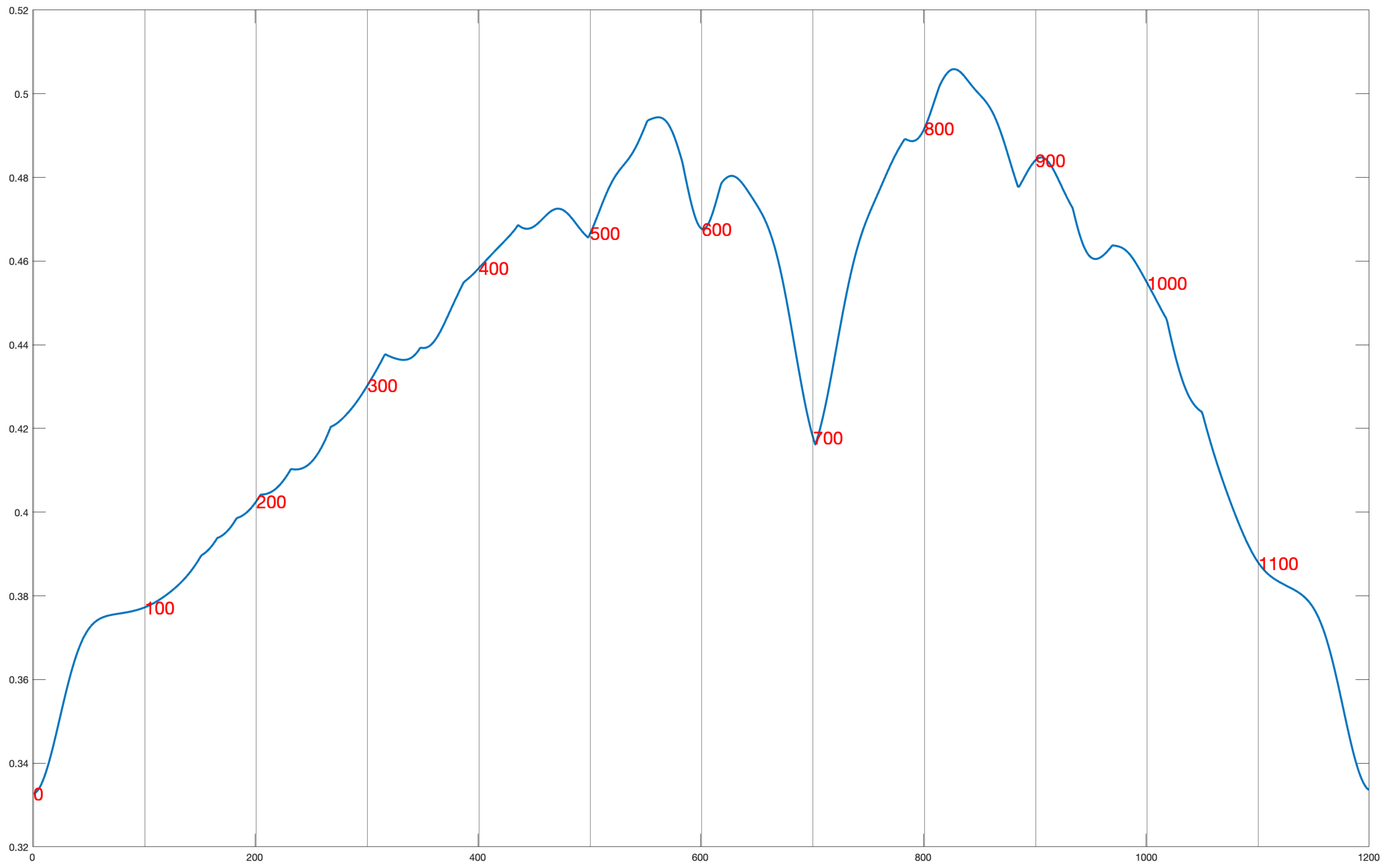
- OIV extends existing formula to account for all overtone interactions
- Histogram of where energy falls
- Can build 1200x1200 matrix, perform PCA
- PC1 ~ dissonance
- But so much more information, with no clear interpretation

Entropy of overtone interaction vector

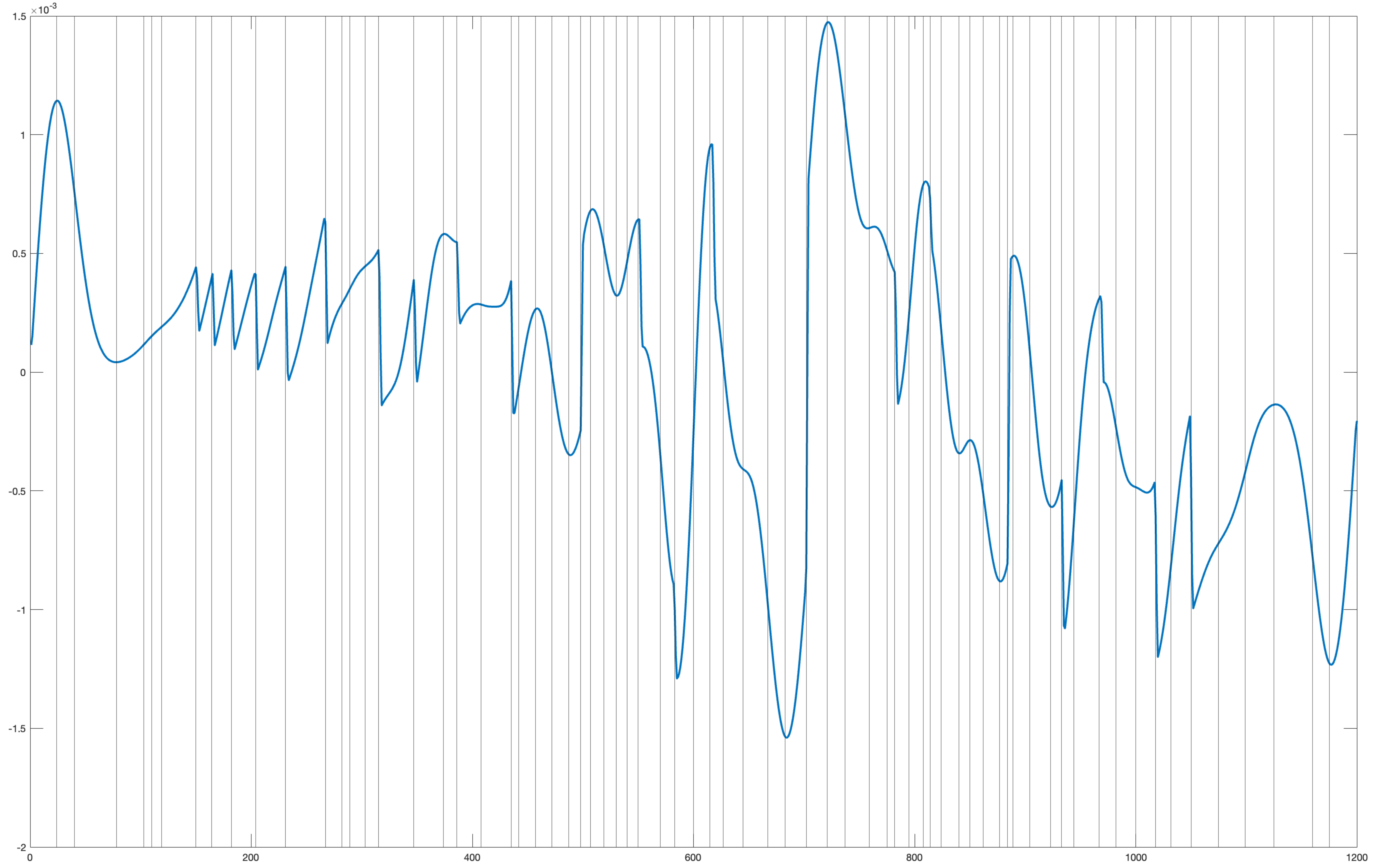
$$H(\Delta) = -\sum_{n} p_{\Delta}(n) \log p_{\Delta}(n)$$



Musically intuitive?



Critical points (discontinuities, maxima, minima) of gradient and gradient^2

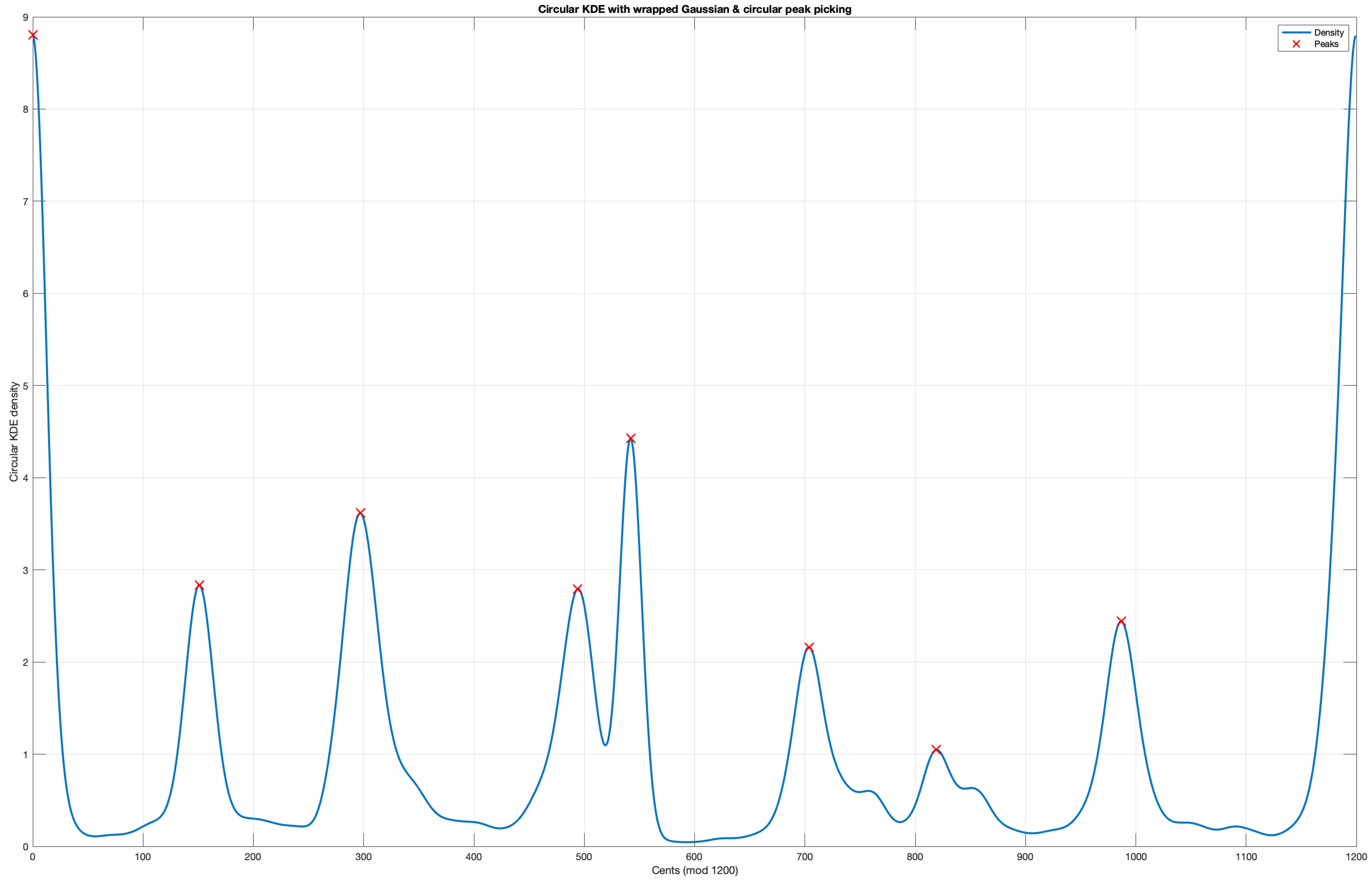


Do performers gravitate to these values?

More than chance?

- 12 Maqamat, 12 Ragas (list, audio, pitch data in appendix)
- Analyze recordings using CREPE
- Compute KDE for each recording
- Find peaks (in cents) of all KDEs
- Treat those as “scale degrees”
- Compare with predicted values (critical points), do they fall within +-7 cents?
- Test alignment with Monte Carlo using random peaks

Performance Analysis
KDE peaks

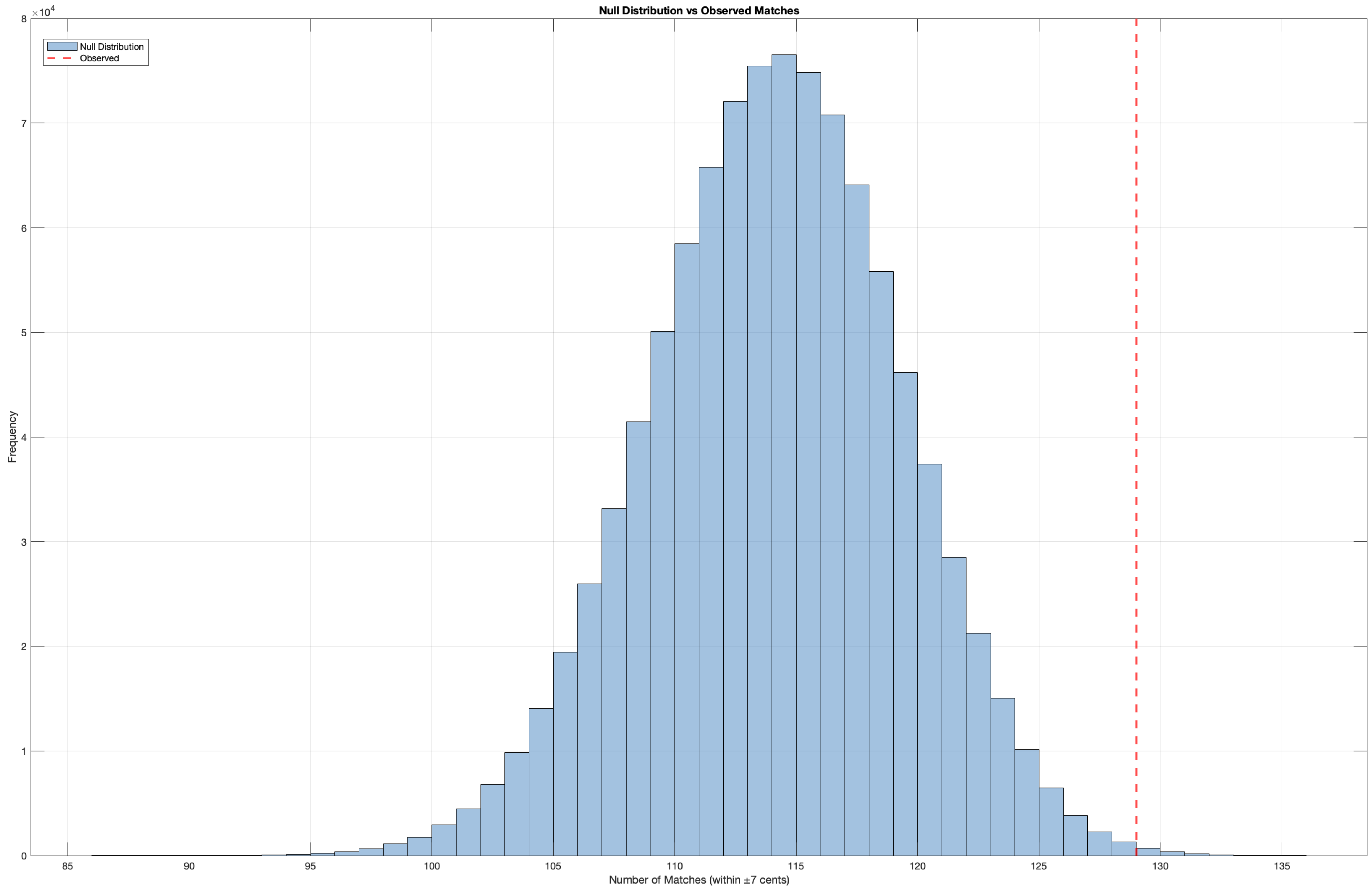


Results

Peaks vs. Critical Points

- 129/149 (87%) peaks fall within +-7 cents of a critical point
- Null: Generating 149 random peaks -> average of 111/149 (74%) fall within +-7 cents
- z-score ~ 3
- p ~ 0.001

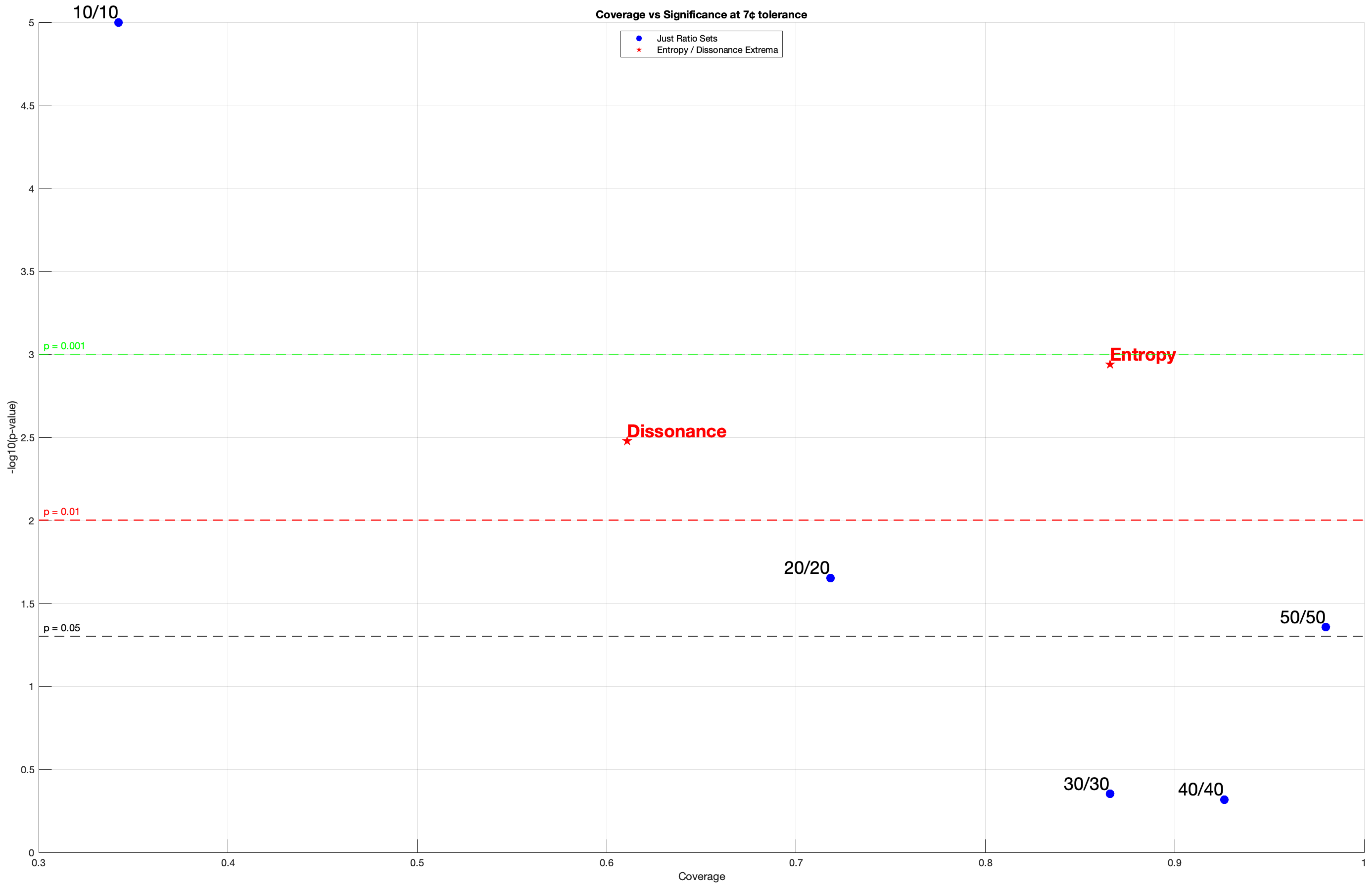
Results of MC test



Just Ratios

- Most critical points ~ just ratios
- But which just ratios?
- Removing simple ratio critical points and peaks (9/8 6/5 5/4 4/3 7/5 3/2 7/4 5/3 9/5)
  - 84/107 (79%), p ~ 0.01
- Consider all ratios P/Q where P,Q <= 10, 20, 30, 40, etc.
  - Mod into octave
  - Treat as critical points
  - Test alignment with peaks +-7 cents
  - Compute coverage (matches / total peaks) and p-value

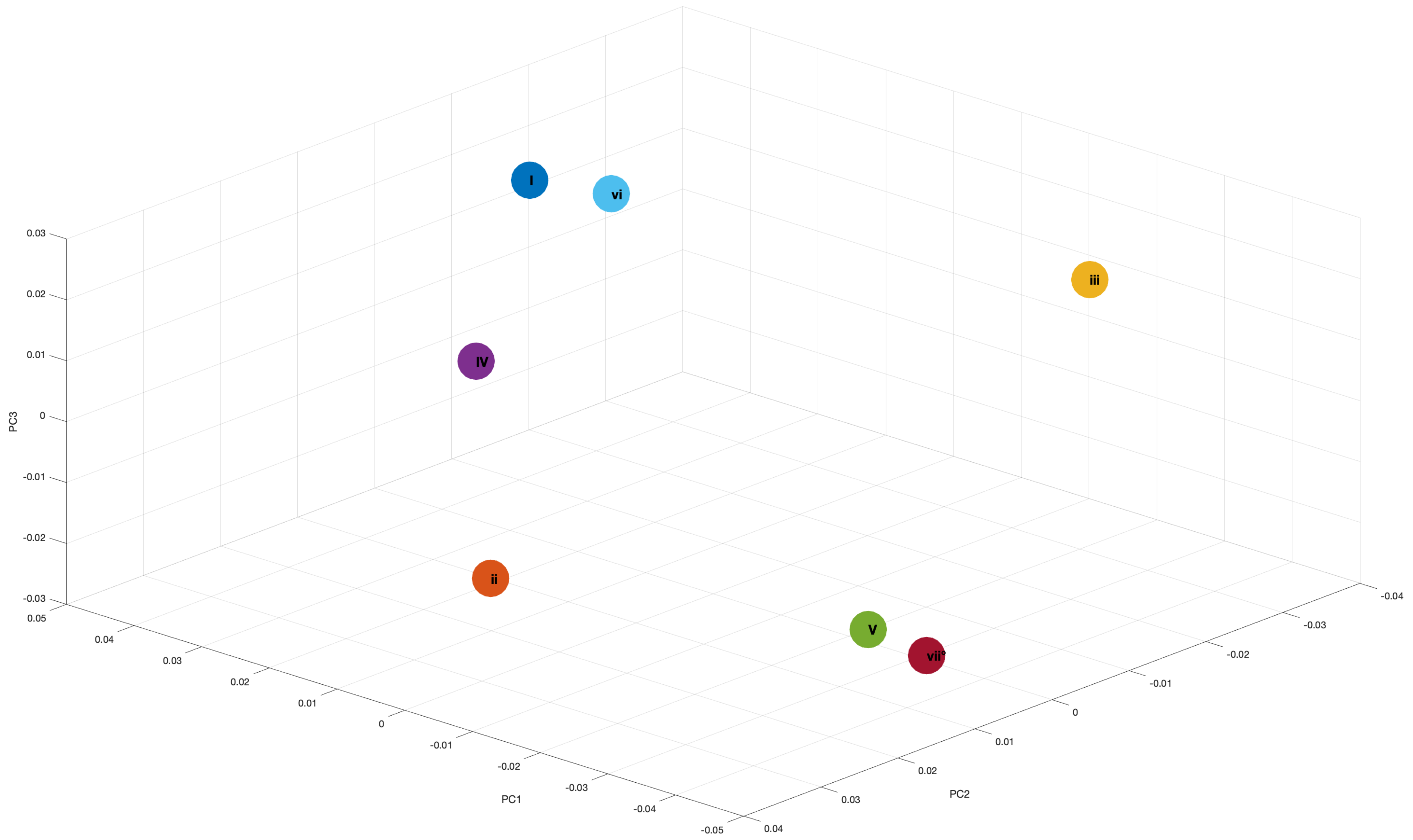
Coverage vs. Significance



Other findings

- Partch Scale
  - 43 points
  - 81/149 (54%) within +-7, $p = 0.083$, $z = 1.39$
- Using gradient and gradient^2 of dissonance
  - Identified 47 critical points
  - 89/149 peaks (61%) within +-7, $p = 0.0032$, $z = 2.75$
- Treating triads as centroids of PC coordinates hints at functional harmony

Other findings - Hint at functional harmony



Conclusion

- OIV framework - generalizes roughness, preserves full overtone interaction
- PCA validation
    - PC1 ~ dissonance, r = 0.9 with Masina et al. (2022)
    - Higher PCs capture other properties (PC6 major/minor quality?)
- Entropy curve - global scalar, predicts critical points as attractors
- Empirical validation - 87% of performed peaks within +-7 cents of extrema vs. 74% expected by chance, p ~ 0.001
- Just ratios can provide same coverage, but at expense of significance

Future work

- Investigate shape of entropy curve around each point / apply thresholding
- Just ratio list generation
- Interpret higher PCs (18 dimensions til elbow point)
- Consider triads, 7th chords (own OIV or centroids) -> functional harmony
- Integrate into technology -> better music analysis, generation
- Different timbres, genres
- Psychoacoustic validation

Thank you!

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Appendix
MATLAB Code

- OIV - calculateOIV.m
- Matrix for PCA - chromatic_matrix.m
- Dissonance - masinaCorr.m
- Entropy - sweepOIVEntropy.m
- Critical Points/Peaks Monte Carlo - empiricalEvidence.m
- KDE - kdestuff.m
- Build just ratio lists / check coverage - floop.m

Appendix
Recordings

- 12 Maqamat
  - Rast, Bayati, Bayati Shuri, Hijazkar, Nahawand, Zanjaran, Hijaz, Shawq Afza, Nahawand Murassaa, Rahat, Suznak, Nawa Athar
- 12 Ragas (Carnatic)
  - Shadvidamargini, Chakravakam, Chitrambari, Kanakangi, Pavani, Dhira Shankarabharanam, Ganamurti, Kokilapriya, Rasikapriya, Mayamalavagowla, Jhankaradhwani, Vanaspati
- All .wav and .csv (from CREPE) available as [lowercase title].*