Proposal

Title

Structural Complexity through Musical Entropy: Six Analytical Perspectives on Beethoven's Late Piano Sonatas (in Comparison with Early and Middle Works)

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1. Abstract

This proposal introduces a research topic based on musical entropy. The core aim is to apply information-theoretic methods to quantify and compare the structural complexity of Beethoven's late piano sonatas with excerpts from his early and middle periods. Beethoven's late works are often described as embodying philosophical depth and artistic complexity. Traditional analyses emphasize features such as motivic development, contrapuntal density, tonal ambiguity, variation techniques, rhythmic freedom, and religious implications. These descriptions, however, are primarily qualitative.

This study proposes to use entropy as a quantitative indicator across six perspectives, in order to explore how Beethoven's late works negotiate the tension between order and complexity. Methodologically, MIDI files are imported into Python to generate a structured notes table (measure, note index, start beat, end beat, pitch name, octave, MIDI value, duration), which will serve as the basis for all entropy calculations:

- From Exposition to Development: Late developments often exhibit tonal ambiguity and freedom. Tonal Shannon entropy will measure concentration versus dispersion, while relative entropy (KL divergence) will quantify the divergence between the development and the exposition.
- Fugue and Counterpoint: Late fugues (e.g., Op.110, Op.106) show complex inter-voice relations. Relative entropy will compare distributions among voices, while network entropy will capture the structural complexity of imitation and contrapuntal interaction.
- Arias and Ambiguous "Romantic" Passages: Singing and blurred sections (e.g., Op.110 Adagio) will be analyzed with rhythmic entropy and conditional entropy to examine freedom and unpredictability.
- Variation Movements: Late variation movements (e.g., Op.111, second movement) highlight multilevel transformation. Relative entropy will quantify the distance between variations and the theme, while Markov entropy rate will examine long-term predictability.
- Dance/Scherzo Movements: Dance-like elements, often distorted in late works, will be analyzed with rhythmic entropy and permutation entropy to measure rhythmic diversity and complexity.
- Chorale/Religious Allusions: Some late sections (e.g., closing passages of Op.110) exhibit solemnity and concentration. These will be associated with low-entropy

states (lower Shannon entropy) reflecting stability and order.

By computing entropy in these six perspectives, and comparing late works with earlier and middle-period excerpts, the project seeks to provide a new, quantitative approach to the complexity of Beethoven's late music.

2. Basic Entropy Formulas (Overview)

- Shannon Entropy

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i)$$

- Relative Entropy / Kullback–Leibler Divergence

$$D_{KL}(P \parallel Q) = \sum_{i=1}^{n} p(x_i) \log \frac{p(x_i)}{q(x_i)}$$

- Conditional Entropy

$$H(Y|X) = -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log p(y|x)$$

- Markov Entropy Rate

$$H = \lim_{n \to \infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n)$$

- Permutation Entropy

$$H_{perm} = -\sum_{i=1}^{k} p(\pi_i) \log p(\pi_i)$$

- Network Entropy

$$H(G) = -\sum_{i=1}^{N} p(k_i) \log p(k_i)$$

3. Research Background

- **Shannon entropy in early music studies:** Hiller (1959) first introduced information theory into music, using entropy to measure material complexity. Meyer (1989) later connected musical "expectation/uncertainty" with entropy, establishing a theoretical foundation.
- **Conditional entropy and predictability (including Beethoven):** Daikoku (2019, PLOS ONE) compared Beethoven's piano sonatas with Bach's Well-Tempered Clavier, showing differences in predictability between melody and bass using conditional entropy.
- **Relative entropy for style/section comparison:** Paz et al. (2022, EMR) applied KL divergence to compare completions of Bach's unfinished fugue, demonstrating how stylistic distance can be quantified.

- **Rhythmic entropy and perception:** De Fleurian et al. (2017, Cognitive Science) showed that rhythmic entropy correlates with human judgments of complexity.
- **Permutation entropy in rhythm:** Bandt & Pompe (2002, PRL) introduced permutation entropy, later applied to musical rhythm for capturing local temporal complexity.
- **Network entropy for counterpoint:** Bell (2016, ICCC) modeled music as networks and calculated entropy to assess structural diversity in polyphonic settings.
- **Expectation and information dynamics:** Pearce & Wiggins (2012, Cognitive Science) proposed the IDyOM framework, linking entropy and information content to listener prediction. Temperley (2019, MTO) introduced the Uniform Information Density hypothesis, suggesting entropy curves may reflect musical pacing and balance.

4. Problem Statements

Problem 1 (Voice Separation after MIDI-to-Table Extraction)
After parsing a MIDI file with Python, we obtain a single table that lists all notes (measure, note index, start beat, end beat, pitch name, octave, MIDI value, duration). For fugues and polyphonic music, however, the notes need to be divided into separate voices (usually three or four) for meaningful entropy analysis. The question is: how should these voices be separated? Should we rely on the voice/track information already in the MIDI file, use rules based on pitch range and onset timing, follow the continuity of voice-leading, or combine these methods? Furthermore, what standards can ensure consistency across different movements and works?

Problem 2 (Significance)
Why are entropy-based measures meaningful for musicology and performance? Traditional analysis already explains many phenomena, so does entropy truly add value? Can it capture dimensions like “complex vs. clear” or “free vs. ordered”? These questions remain open.

Problem 3 (Application of Results)
Entropy values or curves are just numbers—how can they become useful? How might they be connected to listening perception, performance choices, or traditional analysis? This remains to be explored.

Problem 4 (Performance MIDI vs. Score MIDI)
When MIDI files are generated directly from scores, the timing is perfectly aligned with the notation, which is useful for analyzing notated structures. However, if MIDI files are recorded from human performances, there will be expressive variations — such as timing deviations, rubato, dynamics, and articulation — that reflect

interpretive choices. If we use the latter type of data, how could entropy be applied to analyze performance states and expressive variation, rather than just the score?

5. Data

- IMSLP public-domain scores (Late sonatas: Op.101, 106, 109, 110, 111; Early/middle-period excerpts for comparison).

https://vmirror.imslp.org/files/imglnks/usimg/6/6b/IMSLP51800-PMLP01485-Beethoven_Werke_Breitkopf_Serie_16_No_151_Op_101.pdf
https://vmirror.imslp.org/files/imglnks/usimg/8/89/IMSLP728756-PMLP1486-Beeth_op106_EB4342.pdf
https://s9.imslp.org/files/imglnks/usimg/5/51/IMSLP51804-PMLP01487-Beethoven_Werke_Breitkopf_Serie_16_No_153_Op_109.pdf
https://ks15.imslp.org/files/imglnks/usimg/a/a3/IMSLP51805-PMLP01488-Beethoven_Werke_Breitkopf_Serie_16_No_154_Op_110.pdf
https://s9.imslp.org/files/imglnks/usimg/7/7e/IMSLP51811-PMLP01489-Beethoven_Werke_Breitkopf_Serie_16_No_155_Op_111.pdf

- MIDI files and corresponding tables (generated via Python): For each piece, MIDI data are aligned with a structured table that includes:
  1. Measure number
  2. Note index
  3. Start beat
  4. End beat
  5. Pitch name
  6. Octave
  7. MIDI value
  8. Duration

As an example, for Beethoven's Piano Sonata No.32 in C minor, Op.111 (first movement), one MIDI file and one corresponding table are provided.
https://kern.humdrum.org/cgi-bin/ksdata?l=users/craig/classical/beethoven/piano/sonata&file=sonata32-1.krn&f=midi

https://www.kdocs.cn/l/csgLHDQwVVKd

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