Dynamic Programming

Exercises

1. Consider a modification of the rod-cutting problem in which, in addition to a price for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Give a dynamic programming algorithm to solve this modified problem.

2. Modify MEMOIZED-CUT-ROD to return not only the value but the actual solution too.

3. The Fibonacci series begins with 0, 1. Each subsequent number in the series is the sum of the previous two. Give an $O(n)$ time algorithm to compute the nth Fibonnaci number. Draw the subproblem graph. How many vertices and edges are there in the graph? How does this relate to the running time?

4. Explain why memoization fails to speed up the MERGE-SORT algorithm (see Introduction to Algorithms lecture).

5. Suppose that in the rod-cutting problem, we also had a limit, l_i, on the number of pieces of length i we are allowed to produce for $i = 1, 2, \ldots n$. Show that the optimal substructure property no longer holds.

6. Determine an LCS of 10010101 and 010110110.

7. Give a memoized version of LCS-LENGTH that runs in $O(mn)$ time.