Tonal Scales and Minimal Simple Pitch Class Cycles

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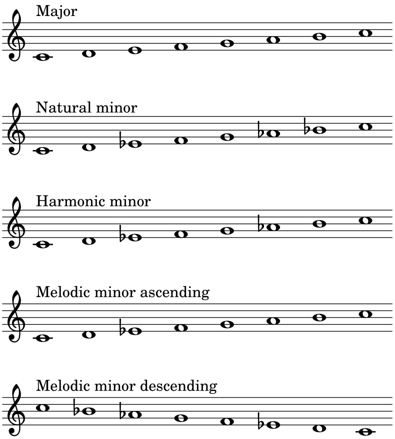
Piston's (1991) "fundamental diatonic scales", $\mathcal{F}$



Piston's (1991) "fundamental diatonic scales", $\mathcal{F}$

Piston's (1991) "fundamental diatonic scales" with their associated translational and inversional equivalence classes of pitch class set:

Common name	*Pitch class set* (P)	$[P]_{\mathrm{T}}$	$[P]_{\mathrm{I}}$
Major	$\{0, 2, 4, 5, 7, 9, 11\}$	$\mathcal{D}$	$\mathcal{D}$
Natural minor	$\{9, 11, 0, 2, 4, 5, 7\}$	$\mathcal{D}$	$\mathcal{D}$
Harmonic minor	$\{0, 2, 3, 5, 7, 8, 11\}$	$\mathcal{H}_{\mathsf{min}}$	$\mathcal{H}$
Melodic minor ascending	$\{0, 2, 3, 5, 7, 9, 11\}$	$\mathcal{M}$	$\mathcal{M}$
Melodic minor descending	$\{0, 2, 3, 5, 7, 8, 10\}$	$\mathcal{D}$	$\mathcal{D}$

$$\mathcal{F} = \mathcal{D} \cup \mathcal{H}_{\mathsf{min}} \cup \mathcal{M}$$

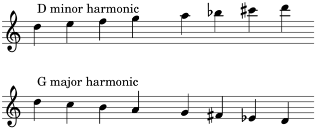
where

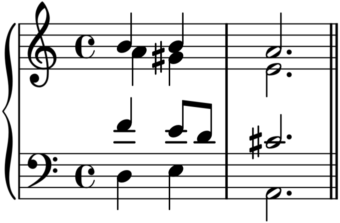
$$\begin{aligned}
\mathcal{D} &= [\{0, 1, 3, 5, 6, 8, 10\}]_{\mathrm{T}} = [\{0, 1, 3, 5, 6, 8, 10\}]_{\mathrm{I}} \quad (7\text{-}35) \\
\mathcal{H}_{\mathsf{min}} &= [\{0, 1, 3, 4, 6, 8, 9\}]_{\mathrm{T}} \quad (\subset 7\text{-}32) \\
\mathcal{H} &= [\{0, 1, 3, 4, 6, 8, 9\}]_{\mathrm{I}} \quad (7\text{-}32) \\
\mathcal{M} &= [\{0, 1, 3, 4, 6, 8, 10\}]_{\mathrm{T}} = [\{0, 1, 3, 4, 6, 8, 10\}]_{\mathrm{I}} \quad (7\text{-}34)
\end{aligned}$$

Minor scales ($\mathcal{H}_{\text{min}}$, $\mathcal{M}$) neglected in diatonic set theory

- Harmonic minor scale sets ($\mathcal{H}_{\text{min}}$) and ascending melodic minor scale sets ($\mathcal{M}$) do not possess many of the mathematical properties of the diatonic sets ($\mathcal{D}$), identified in the literature
- For example, none of the eight properties of the diatonic set identified by Clough, Engebretsen & Kochavi (1999) are possessed by the usual forms of the minor scales
- Focus here is on some mathematical properties that are shared by all *and only* the sets associated with the major *and minor* scales

Major and minor scales and their inversions



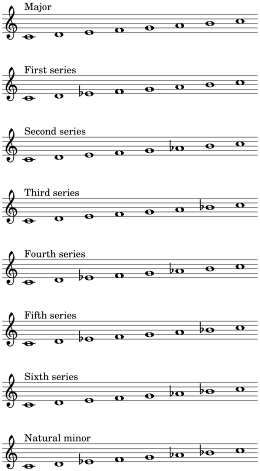


- Inversion of a diatonic set ($\mathcal{D}$) is a diatonic set
- Inversion of an ascending melodic minor scale set ($\mathcal{M}$) is an ascending melodic minor scale set
- Inversion of a harmonic minor scale set ($\mathcal{H}_{\text{min}}$) is a "major harmonic scale" set (Rahn, 1991)
  - $\mathcal{H}_{\mathrm{maj}} = [\{0, 1, 3, 5, 6, 8, 9\}]_{\mathrm{T}}$
  - Also known as the "Moll-Dur-Tonart" (Hauptmann, 1853) and the "second series" in Schenker's (1954) "products of combination"
  - Commonly occurs in cases of the Picardy third (see example on left)

The set of tonal scale sets, $\mathcal{T}$

- Let $\mathcal{H} = [\{0, 1, 3, 4, 6, 8, 9\}]_{\mathrm{I}}$ (Forte's (1973) set class 7-32)
- Call $\mathcal{H}$ the *set of harmonic scale sets*, $\mathcal{H} = \mathcal{H}_{\mathrm{min}} \cup \mathcal{H}_{\mathrm{maj}}$
- Smallest inversionally closed superset of $\mathcal{F}$ is $\mathcal{F} \cup \mathcal{H}_{\mathrm{maj}}$
- Let $\mathcal{T} = \mathcal{F} \cup \mathcal{H}_{\mathrm{maj}} = \mathcal{D} \cup \mathcal{M} \cup \mathcal{H}_{\mathsf{min}} \cup \mathcal{H}_{\mathrm{maj}} = \mathcal{D} \cup \mathcal{M} \cup \mathcal{H}$
- Call $\mathcal{T}$ the *set of tonal scale sets*

Tonal scale sets, $\mathcal{T}$, and Schenker's (1954) products of combination



- Schenker's six "series" generated from the major scale by flattening one or more of the 3rd, 6th and 7th degrees
- Call these six "products of combination", along with the major and natural minor scales from which they are generated, *Schenker's combination scales*
- The class of pitch class sets associated with Schenker's combination scales is equal to the set of tonal scale sets, $\mathcal{T}$

Tonal scale sets, $\mathcal{T}$, and Schenker's (1954) products of combination

Triads			*Scale with C as tonic*	$[P]_\mathrm{T}$	$[P]_\mathrm{I}$	*Series*
I	**IV**	**V**	**C D E F G A B C**	$\boldsymbol{\mathcal{D}}$	$\boldsymbol{\mathcal{D}}$	—
I	IV	v	C D E F G A B♭ C	$\mathcal{D}$	$\mathcal{D}$	Third
I	**iv**	**V**	**C D E F G A♭ B C**	$\boldsymbol{\mathcal{H}_{\mathrm{maj}}}$	$\boldsymbol{\mathcal{H}}$	**Second**
I	iv	v	C D E F G A♭ B♭ C	$\mathcal{M}$	$\mathcal{M}$	Sixth
i	**IV**	**V**	**C D E♭ F G A B C**	$\boldsymbol{\mathcal{M}}$	$\boldsymbol{\mathcal{M}}$	**First**
i	IV	v	C D E♭ F G A B♭ C	$\mathcal{D}$	$\mathcal{D}$	Fifth
i	**iv**	**V**	**C D E♭ F G A♭ B C**	$\boldsymbol{\mathcal{H}_{\mathrm{min}}}$	$\boldsymbol{\mathcal{H}}$	**Fourth**
i	iv	v	C D E♭ F G A♭ B♭ C	$\mathcal{D}$	$\mathcal{D}$	—

- Scales with tonic C, generated by combining tonic, dominant and subdominant triads that can be either major or minor.
- $\mathcal{T}$ is generated by all 8 combinations as well as by only those combinations containing V (in bold)

Pitch class sequences

- A *pitch class sequence* is an ordered set of pitch classes
  - e.g., $\langle 0, 2, 4, 5, 7, 9, 11, 0 \rangle$ represents one octave of an ascending C major scale
- If **S** is a pitch class sequence, then $\mathrm{P}(\mathbf{S})$ is the *associated pitch class set* of **S** and contains all and only the pitch classes that occur in **S**
  - e.g., if $\mathbf{S} = \langle 0, 2, 4, 2, 4, 0, 2 \rangle$ then $\mathrm{P}(\mathbf{S}) = \{0, 2, 4\}$
- If **S** is a pitch class sequence, then $\mathrm{I}(\mathbf{S})$ is the *interval set* of **S** and contains the pitch class intervals that occur between *consecutive* elements in **S**
  - e.g., $\mathrm{I}(\langle 0, 2, 4, 0 \rangle) = \{2, 8\}$, $\mathrm{I}(\langle 0, 4, 7, 11, 2, 5, 9, 0 \rangle) = \{3, 4\}$

Pitch class cycles

- A *pitch class cycle* is a pitch class sequence that begins and ends on the same pitch class.
  - e.g., $\langle 0, 2, 4, 0 \rangle$ is a pitch class cycle but $\langle 0, 2, 4 \rangle$ isn't
- A *simple pitch class cycle* is a pitch class cycle that contains no element more than once except the first element which occurs only at the beginning and at the end
  - $\langle 0, 2, 4, 2, 0 \rangle$ is a pitch class cycle but it is *not* simple because 2 occurs twice, and not only at the beginning and the end
  - $\langle 0, 2, 4, 0 \rangle$ and $\langle 0, 4, 7, 11, 2, 5, 9, 0 \rangle$ are simple pitch class cycles

Minimal simple pitch class cycles

- A simple pitch class cycle is *minimal* if there is no shorter pitch class cycle that has the same interval set
  - e.g., $\langle 0, 4, 7, 11, 2, 5, 9, 0 \rangle$ is minimal because there exists no shorter pitch class cycle that has the interval set $\{3, 4\}$
- $\Gamma(I)$ denotes the *set of minimal simple cycles* that have the interval set I
- $\mathcal{C}(I)$, the *set of minimal simple cycle sets* for interval set I, contains those pitch class sets associated with cycles in $\Gamma(I)$
  - e.g., $\mathcal{C}(\{2, 3\}) = [\{0, 2, 4, 6, 9\}]_{\mathrm{I}} \cup [\{0, 2, 4, 7, 9\}]_{\mathrm{I}}$

Tonal scale sets, $\mathcal{T}$, and minimal simple pitch class cycles

$$\mathcal{T} = \mathcal{C}(\{3, 4\})$$

- The set of tonal scale sets is equal to the set of minimal simple cycle sets for the interval set $\{3, 4\}$
- Note that this is a simple mathematical property that is possessed by all *and only* those pitch class sets associated with the major *and minor* scales recognized in traditional tonal theory
- It contrasts with properties such as the ones considered by Clough, Engebretsen & Kochavi (1999) which apply only to the diatonic sets
- It shows how the traditional major and minor scales can be generated elegantly from the consonant third intervals on which tonal harmony is based

Minimal simple cycle sets and interval set inversion

Theorem

If I is a pitch class interval set, then $\mathcal{C}(I) = \mathcal{C}(\mathrm{INV}(I))$ where $\mathrm{INV}(I)$ is the inversion of I.

It follows that

$$\mathcal{T} = \mathcal{C}(\{8, 9\})$$

Spectra

- If P is a pitch class set, then let $e_k(P)$ denote the *k-spectrum* of P (as defined by Clough & Myerson (1985) and often notated $< k >$)
- If $S \subseteq [P]_{\mathrm{T}}$, then the k-spectrum of every set in S is the same for a given value of k
- We can therefore talk about the k-spectrum of a class of sets, S, provided that S is a subset of a transpositional equivalence class

Minimal simple cycle sets for the diatonic 2-spectrum

- We can therefore say that

$$\mathcal{T} = \mathcal{C}(e_2(\mathcal{D})) = \mathcal{C}(e_5(\mathcal{D}))$$

- That is, the set of tonal scale sets is equal to the set of minimal simple cycle sets for the diatonic 2-spectrum (and the diatonic 5-spectrum)
- This shows how all and only the traditional Western tonal scales can be elegantly generated from the diatonic set

Minimal simple cycle sets for the diatonic spectra

k	$e_k(\mathcal{D})$	$\mathcal{C}(e_k(\mathcal{D}))$
1	$\{1, 2\}$	$\mathcal{D} \cup \mathcal{M} \cup \mathcal{W}$
2	$\{3, 4\}$	$\mathcal{D} \cup \mathcal{M} \cup \mathcal{H}$
3	$\{5, 6\}$	$\mathcal{D}$
4	$\{6, 7\}$	$\mathcal{D}$
5	$\{8, 9\}$	$\mathcal{D} \cup \mathcal{M} \cup \mathcal{H}$
6	$\{10, 11\}$	$\mathcal{D} \cup \mathcal{M} \cup \mathcal{W}$

where

$$\mathcal{D} = [\{0, 1, 3, 5, 6, 8, 10\}]_{\mathrm{I}} \qquad (7\text{-}35)$$

$$\mathcal{M} = [\{0, 1, 3, 4, 6, 8, 10\}]_{\mathrm{I}} \qquad (7\text{-}34)$$

$$\mathcal{H} = [\{0, 1, 3, 4, 6, 8, 9\}]_{\mathrm{I}} \qquad (7\text{-}32)$$

$$\mathcal{W} = [\{0, 1, 2, 4, 6, 8, 10\}]_{\mathrm{I}} \qquad (7\text{-}33)$$

Spectrum set

- If $S \subseteq [P]_{\mathrm{T}}$ for a pitch class set, P, then the *spectrum set* of S, denoted by $\mathrm{E}(S)$, is $\{e_k(S) \mid 1 \leq k < n\}$ where n is the cardinality of a pitch class set in S
- $\mathrm{E}(S)$ is therefore the set of spectra for a pitch class set in S
- For example,

$$\mathrm{E}(\mathcal{D}) = \{\{1,2\},\{3,4\},\{5,6\},\{6,7\},\{8,9\},\{10,11\}\}$$

Minimal simple cycle sets for the diatonic spectra

- If $S \subseteq [P]_{\mathrm{T}}$ for some specified pitch class set, P, then let

$$\mathcal{C}(S) = \bigcup_{e \in \mathrm{E}(S)} \mathcal{C}(e)$$

- That is, $\mathcal{C}(S)$ is the union of the sets of minimal simple cycle sets for the spectra of a pitch class set in S
- Therefore

$$\mathcal{C}(\mathcal{D}) = \mathcal{D} \cup \mathcal{M} \cup \mathcal{H} \cup \mathcal{W} = \mathcal{T} \cup \mathcal{W}$$

where $\mathcal{W}$ is what Rahn (1991) calls the "whole-tone scale with filler tone"

$\mathcal{C}(\mathcal{D})$ and contradictions

- Let k and ℓ be two generic intervals between the members of a pitch class set P and let κ and λ be the specific intervals corresponding to k and ℓ, respectively
- If k is less than ℓ and κ is greater than λ, then this is a case of *contradiction* (Clough & Douthett, 1991; Rahn, 1991)
- Let a *non-contradictory* set be one that has no contradictions
- Let $\mathcal{R}_{\mu,n}$ denote the set of non-contradictory sets of cardinality n when the chromatic cardinality is μ
- Rahn (1991) observes that $\mathcal{R}_{12,7} = \mathcal{D} \cup \mathcal{M} \cup \mathcal{H} \cup \mathcal{W}$
- From the result on the previous slide, it follows that

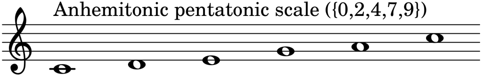
$$\mathcal{C}(\mathcal{D}) = \mathcal{R}_{12,7}$$

- That is, a set is a minimal simple cycle set for a diatonic spectrum if and only if it is a non-contradictory set of cardinality 7

Contiguous-pair interval sets

- A *contiguous* interval set is one of the form $\{i | j \leq i < k\}$ for some specified values of j and k ($j < k$)
- Each diatonic spectrum is a contiguous-pair interval set of the form $\{i, i+1\}$ where $i \in \{1, 3, 5, 6, 8, 10\}$
  - Note that if we interpret this interval set as a pc set, we have a diatonic set without 0!
  - The set of diatonic sets is the intersection of the minimal simple cycle sets for all the diatonic spectra...
- Each diatonic spectrum has an interesting set of minimal simple cycle sets
- So what about the minimal simple cycle sets for the other contiguous-pair interval sets?
- These are $\{i, i+1\}$ where $i \notin \{1, 3, 5, 6, 8, 10\}$ — that is, $i \in \{2, 4, 7, 9\}$)

The pentatonic scale sets and the pentatonic spectra



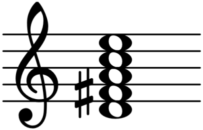
$$\{\{i, i{+}1\} \mid i \in \{2,4,7,9\}\} = \{\{2,3\},\{4,5\},\{7,8\},\{9,10\}\} = \mathrm{E}(\mathcal{P})$$

where $\mathcal{P} = [\{0,2,4,7,9\}]_{\mathrm{T}} = [\{0,2,4,7,9\}]_{\mathrm{I}}$

Let's call $\mathcal{P}$ be the *set of pentatonic scale sets* and $\mathrm{E}(\mathcal{P})$ the *set of pentatonic spectra*

Minimal simple cycle sets for the pentatonic spectra

Maj. dom. 9th



k	$e_k(\mathcal{P})$	$\mathcal{C}(e_k(\mathcal{P}))$
1	$\{2,3\}$	$\mathcal{P} \cup \mathcal{N}$
2	$\{4,5\}$	$\mathcal{P}$
3	$\{7,8\}$	$\mathcal{P}$
4	$\{9,10\}$	$\mathcal{P} \cup \mathcal{N}$

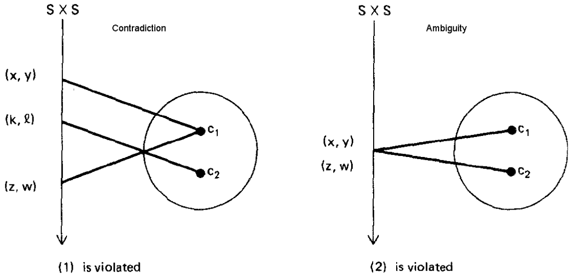
$$\mathcal{P} = [\{0,2,4,7,9\}]_{\mathrm{T}} = [\{0,2,4,7,9\}]_{\mathrm{I}} \qquad (5\text{-}35)$$

$$\mathcal{N} = [\{0,2,4,6,9\}]_{\mathrm{T}} = [\{0,2,4,6,9\}]_{\mathrm{I}} \qquad (5\text{-}34)$$

$$\mathcal{C}(\mathcal{P}) = \mathcal{P} \cup \mathcal{N}$$

- $\mathcal{N}$ is the class of sets associated with the complete major dominant ninth chords (see above)
- $\mathcal{N}$'s rôle with respect to $\mathcal{P}$ is analogous to the rôle of the minor scale sets $(\mathcal{M}, \mathcal{H}, \mathcal{W})$ with respect to the diatonic scale sets $(\mathcal{D})$
- $\mathcal{N}$ is the set complement of $\mathcal{M}$; $\mathcal{P}$ is the complement of $\mathcal{D}$

Rothenberg's (1978a) concept of propriety



From Rothenberg (1978a, p. 206)

(hj) is an interval between two tones on a continuous frequency scale, S

$F(hj)$ maps interval $(hj) \in S \times S$ onto a discrete point called the *code*

$$(F(xy) = F(zw)) \wedge ((xy) < (kl) < (zw)) \Rightarrow (F(kl) = F(xy)) \quad (1)$$

$$(xy) \sim (zw) \Rightarrow F(xy) = F(zw) \quad (2)$$

Proper if (1) satisfied, strictly proper if (1) and (2) satisfied

Rothenberg's (1978b) concepts of stability and efficiency

- Let P be the range of F, i.e., the set of discrete code points onto which intervals between frequencies in S are mapped
- *Stability* of P is the proportion of undirected intervals in P that are *not* ambiguous
    - e.g., stability of a diatonic set is $\frac{20}{21}$
- Q is a *sufficient set* for P iff P is the only member of $[P]_{\mathrm{T}}$ that contains Q
- *Efficiency* of a set is the average size of a sufficient set for P as a proportion of the size of P
    - Indicates how much of a set needs to be presented on average in order for it to be distinguished from all of its transpositions

T, $\mathcal{C}(\mathcal{D})$, $\mathcal{C}(\mathcal{P})$, propriety, stability and efficiency

Proper 5- and 7-note sets when $\mu = 12$, ordered by increasing cardinality, then decreasing stability, then decreasing efficiency

P	$[P]_\mathrm{I}$	Stability	Efficiency
$\{0,2,4,7,9\}$	$\mathcal{P} = [\{0,2,4,7,9\}]_\mathrm{I}$	1.0000	0.8000
$\{0,2,4,6,9\}$	$\mathcal{N} = [\{0,2,4,6,9\}]_\mathrm{I}$	0.9000	0.6400
$\{0,1,4,6,9\}$	$[\{0,1,4,6,9\}]_\mathrm{I}$	0.9000	0.6000
$\{0,1,4,6,8\}$	$[\{0,1,4,6,8\}]_\mathrm{I}$	0.6000	0.5800
$\{0,2,4,6,8\}$	$[\{0,2,4,6,8\}]_\mathrm{I}$	0.4000	1.0000
$\{0,1,3,6,9\}$	$[\{0,1,3,6,9\}]_\mathrm{I}$	0.4000	0.6400
$\{0,1,3,5,6,8,10\}$	$\mathcal{D} = [\{0,1,3,5,6,8,10\}]_\mathrm{I}$	0.9542	0.7687
$\{0,1,3,4,6,8,10\}$	$\mathcal{M} = [\{0,1,3,4,6,8,10\}]_\mathrm{I}$	0.7143	0.6299
$\{0,1,3,4,6,8,9\}$	$\mathcal{H} = [\{0,1,3,4,6,8,9\}]_\mathrm{I}$	0.4762	0.6259
$\{0,1,2,4,6,8,10\}$	$\mathcal{W} = [\{0,1,2,4,6,8,10\}]_\mathrm{I}$	0.2857	0.6327

- When ranked in decreasing order of stability, then efficiency, union of two highest-ranked 5-note classes is $\mathcal{C}(\mathcal{P})$
- Class of proper 7-note sets equals $\mathcal{C}(\mathcal{D}) = \mathcal{R}_{12,7}$
- Union of three most stable 7-note classes equals $\mathcal{T}$

$\mathcal{T}$, $\mathcal{C}(\mathcal{P})$ and Carey's (2002) coherence quotient

Carey (2002) defines a set to be *coherent* if it contains no contradictions and no ambiguities

- If S is a set, then coherence quotient, $CQ(S)$ is

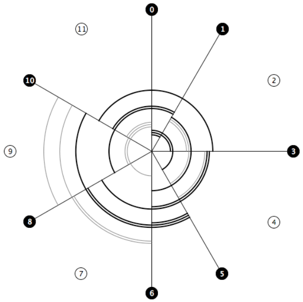
$$\frac{AF(S)}{SF(N)}$$

  where

  - $AF(S)$ is the number of contradictions and ambiguities in S
  - $SF(N)$ is the maximum number of simultaneous coherence failures for a set of cardinality N

- In decreasing order, the three 7-note set classes with the highest coherence quotient are $\mathcal{D}$, $\mathcal{M}$ and $\mathcal{H}$
- That is, $\mathcal{T}$ is the union of the three most coherent set classes, when coherence is measured using Carey's coherence quotient
- The most coherent 5-note class is $\mathcal{P}$; $\mathcal{N}$ is one of the two, second most coherent 5-note classes

A spring-based method for measuring evenness



- Every pitch class connected to every other pitch class by the shortest possible spring
- Resting length of each spring is the length it would have if the set were a perfectly even set with the same cardinality
- Measure the potential energy in each spring, assuming Hooke's law
- Energy in a spring is $((c/\mu) - (d/|P|))^2$, where c and d are the chromatic and diatonic intervals for the spring
- Unevenness of set is sum of potential energies in all springs

$\mathcal{T}$, $\mathcal{C}(\mathcal{P})$ and evenness

- When spring-based method used,
  - $\mathcal{T}$ is the union of the 3 most even 7-note set classes
  - $\mathcal{C}(\mathcal{P})$ is the union of the 2 most even 5-note set classes

When other measures of evenness are used, these results do *not* hold (SBM= Spring-based method; BD= Block & Douthett's (1994) method):

SBM rank	$U(P)$	$[P]_{\mathrm{I}}$	*BD rank*
1	0.0278	$\mathcal{D} = [\{0, 1, 3, 5, 6, 8, 10\}]_{\mathrm{I}}$	1
2	0.0417	$\mathcal{M} = [\{0, 1, 3, 4, 6, 8, 10\}]_{\mathrm{I}}$	2
3	0.0556	$\mathcal{H} = [\{0, 1, 3, 4, 6, 8, 9\}]_{\mathrm{I}}$	4
4=	0.0694	$\mathcal{W} = [\{0, 1, 2, 4, 6, 8, 10\}]_{\mathrm{I}}$	3
4=	0.0694	$[\{0, 1, 2, 4, 6, 8, 9\}]_{\mathrm{I}}$	?

Summary

$\mathcal{T}$ is:

- the smallest inversionally closed superset of $\mathcal{F}$;
- the pitch class sets associated with Schenker's combination scales;
- the pitch class sets generated by combining a major dominant triad with a major or minor tonic triad and a major or minor subdominant triad;
- the pitch class sets generated by combining primary triads that can each be either major or minor;
- the minimal simple cycle sets for the diatonic 2-spectrum, $\{3, 4\}$;
- the union of the three most stable 7-note set classes (using Rothenberg's definitions of stability and propriety);
- the union of the three most coherent 7-note set classes (using Carey's coherence quotient); and
- the union of the three most even 7-note set classes (using the spring-based method for measuring evenness).

Summary

- $\mathcal{C}(\mathcal{D}) = \mathcal{R}_{12,7}$
- $\mathcal{C}(\mathcal{P}) = \mathcal{P} \cup \mathcal{N}$
- $\mathcal{C}(\mathcal{P})$ is
  - the union of the 2 highest-ranked 5-note set classes when sorted by stability and then efficiency
  - a subset of the union of the 5-note set classes with the two highest coherence quotients
  - the union of the two most even 5-note set classes (when evenness measured using the spring-based method)

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