A parallel algorithm for finding maximal transformed matches of polyphonic patterns in unvoiced polyphonic music

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July 11, 2024

We present a parallel algorithm for finding transformed matches of a query pattern in a database of symbolic music encodings in which voice information is absent, ambiguous or unreliable (e.g., an encoding of a score or performance of a keyboard work). Our algorithm allows users to define the class of transformations by which matches may be related to a query pattern. We assume an encoding, D, in the database and the query pattern, P, are represented by sets of k-dimensional points, $D, P \subset \mathbb{R}^k$. In addition to P and D, the algorithm takes as input a user-defined class, F, of transformations, each of which must be a bijection over $\mathbb{R}^k$.

We define Q to be a *maximal match* of P in D with respect to F, if there is an $f \in F$ such that $Q = f(P')$, where $P' \subseteq P$ and there is no S such that $P' \subset S$ and $f(S) \subseteq D$. Our algorithm computes all maximal matches of P in D with respect to F. If $m = |P|$ and $n = |D|$, then the algorithm does $\Theta((mn)^\beta \log n)$ work, has $\Theta(\beta(\log m + \log n))$ span and uses $\Theta((mn)^\beta)$ space, where β is the *basis size* associated with the transformation class F. For example, if F contains the traditional contrapuntal transformations of transposition, augmentation, diminution, inversion, retrograde and their combinations, then $\beta = 2$. We evaluated the algorithm on two musicological tasks: discovering occurrences of the "HAYDN" theme in Ravel's *Menuet sur le nom d'Haydn*, on which the algorithm achieved an F_1 score of 0.70; and discovering subject entries in *Contrapunctus VI* from Bach's *Die Kunst der Fuge*, on which the algorithm achieved an F_1 score of 0.93.