ABSTRACTS


TOWARD A FORMAL COGNITIVE THEORY OF HARMONIC PITCH STRUCTURE IN THE MUSIC OF J.S. BACH

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1. Pitch class simultaneity representations and the aim of the theory.

A. Pitch simultaneity representations.

A pitch simultaneity can be defined as a segment of a score which has constant pitch content throughout its extent and which differs in pitch content from neighbouring segments. A pitch simultaneity can be represented by a pitch set and the location in the score at which the pitch simultaneity begins. A score can therefore be transformed by an effective procedure into a pitch simultaneity representation. Figure 1. (a) shows the pitch simultaneity representation of the example in Figure 1.(b).

B. Pitch class simultaneity representations.

A pitch class simultaneity can be defined as a segment of a score which has constant pitch class content throughout its extent and which differs in pitch class content from neighbouring segments. A pitch class simultaneity can be represented as a pitch class set and the location in the score at which the pitch class simultaneity begins. The pitch class simultaneity representation of a score can be derived by an effective procedure from the pitch simultaneity representation by unifying any adjacent pitch simultaneities with the same pitch class content. Figure 2.(a) shows the pitch class simultaneity representation of the example in Figure 2.(b).

C. The aim of the theory.

The aim of the theory is to devise a formal rule system capable of describing the pitch class simultaneity representations of all and only well-formed Bach Chorales. If this can be achieved, the rule system will constitute a formal theory of pitch class structure in Bach Chorales. If this has really been achieved, it

Figure 1. Pitch simultaneity representations.

Figure 2. Pitch class simultaneity representations.
will be possible to embody the theory in two computer programs:
1. a program capable of analysing any real Chorale in such a way as to show how it is accountable for by the formal rule system for pitch class simultaneity structure; and
2. a program capable of generating all and only well-formed pitch class simultaneity structures of the Chorale type.

II. Path-type pitch class sets.

A. Thirds space.

Thirds space is a two-dimensional space best conceived as covering the surface of a torus. There are 12 discrete points in this space corresponding to the 12 octave equivalence pitch classes in a 12-fold equal-tempered pitch system. Each point in the space has four other points which are proximal to it in the space. For a pitch class, p, these four points represent the pitch classes, (p + 4) mod 12, (p - 4) mod 12, (p + 3) mod 12 and (p - 3) mod 12. Thirds space is isomorphic to the direct product of the two cyclic groups, C3 and C4 (Balzano, 1980).

B. Definition of path-type pitch class sets.

Pitch class sets in thirds space can be conceived in two different ways - one "dynamic" and one "static".

The "static" conception involves describing regions or areas of the space which contain all the pitch classes in the set which it is necessary to represent. This leads to the representation of the diatonic set as a "most compact region" of the space containing seven different pitch classes (Balzano, 1980). When this mode of conception is adopted, the diatonic set has unique properties which it does not share with any other seven-member pitch class set.

The "dynamic" mode of conception involves moving through the space collecting pitch classes. When this mode of representation is adopted, a special type of seven-member pitch class set can be defined. If a path is taken through thirds space such that:
1. exactly one complete circuit is made around the major circumference of the torus;
2. exactly one complete circuit is made around the minor circumference of the torus; and
3. no pitch class is passed through more than once until the starting pitch class is again reached on completion of both circuits described in 1 and 2;
then only four distinct transpositional equivalence types of pitch class set are generated. Pitch class sets which fall into these categories are called path-type sets in this study.

The minor harmonic, minor melodic ascending and major scales represent three of the four transpositional equivalence path-type sets and all of the inversionsal and transpositional equivalence path-type sets. The fourth transpositional equivalence type set can be represented by a "major harmonic" scale, (0,2,4,5,7,8,11), or as an inversion of the minor harmonic scale.

This would therefore seem to provide a formal definition of the category of pitch class sets which can provide the pitch class substrate of all the standard tonal scales which have emerged in music theory from an intuitive study of tonal music.

C. Path-type pitch class sets as pitch class simultaneity transition supersets.

There is no formal or computable theory of how "scales" are manifested in the structure of tonal music although many partial "theories" have emerged (for example, Rameau, 1771; Schenker, 1935/79; Lerdahl and Jackendoff, 1983). On studying the pitch class simultaneity representations of several Bach Chorales it has been possible to posit two hypotheses concerning the manner in which path-type pitch class sets appear to constrain the pitch structure of the style of music represented by the Chorale genre.

If sets S1, S2, S3 and S4 are the sets corresponding to the first four pitch class simultaneities in a piece then for each pitch class simultaneity transition,

\[ S_n \Rightarrow S_{n+1} \]

a pitch class simultaneity transition set can be defined,

\[ TS_n \cap S_n \cup S_{n+1} \]

Each pitch class simultaneity, S_n, will be involved in two such transitions (unless it is the first or last simultaneity in the piece), TS_n and TS_{n-1}. The first path-type set structure hypothesis is that for each pitch class simultaneity, S_n, at least one of TS_n, TS_{n-1} will be a subset of at least one path-type set. The second path-type set structure hypothesis is that it is possible to interpret the path-type set structure of a well-formed Chorale in such a way that it is never necessary to change the operational path-type set by more than one pitch class.
III. Harmonic cores and their roots.

A. Well-formed harmonic cores and their roots.
   It is useful to define a well-formed harmonic core type set as a set which is
   transpositionally or inversonally equivalent to the pitch class set \((C,E\,flat,\,G)\) - that is, all
   major and minor triads. It is also useful to
   define the root of an harmonic core so that in
   the transpositional equivalence classes
   represented by the sets, \((0,3,7)\) and \((0,4,7)\) the
   root in each case is 0.

B. Three levels of pitch class simultaneity superset.
   It is useful to define three levels of pitch
   class simultaneity superset:
   1. the pitch class simultaneity set itself
      (simultaneity set, S);
   2. the smallest connected region along the
      operational path-type set which contains the
      pitch class simultaneity set (smallest
      connected superset, SCS); and
   3. if the SCS contains less than two harmonic
      cores, a number of implied supersets
      forming connected regions along the
      operational path-type set which are
      formable from the SCS by extension of it
      by only one member along the operational
      path-type set in either direction and only
      then if by doing so, the number of
      harmonic cores in the new set is greater
      than in the SCS (extended connected
      superset, ECS).

C. The implied harmonic cores of a pitch class
   simultaneity set.
   For each level of pitch class simultaneity superset, a number of implied harmonic cores
   can be defined. These are those core-type subsets of the pitch class simultaneity superset
   at any one of the three levels. A loose
   hypothesis can be formulated as follows: that the "stability" of a pitch class simultaneity is
   inversely related to the number of harmonic cores which are strongly implied by the pitch

class simultaneity set. Another loose
hypostasis can be formulated as follows: that
for a given S, the harmonic cores implied by
the S are more strongly implied than those
implied by the possible SCSs which are in turn
more strongly implied than those implied by the
possible ECSs.

D. The need for well-formedness rules for the
   harmonic core structure of Bach Chorales.
   It is my belief that the foregoing path-type
   set structure theory needs to be supplemented
   by a system of rules constraining the harmonic
   core root structure of a Chorale in order that the
   aim of the theory may be achieved. I have
   devised several such rule systems and tested
   them on real Chorales with varying success. In
   two cases, a complete harmonic core root
   structure theory has been formulated which in
   combination with the foregoing path-type set
   structure theory is capable of providing a
   complete generative pathway to the pitch class
   simultaneity structure of all the test Chorales.
   However, earlier stages in this generative
   pathway do not seem to give rise necessarily to
   perceptually more salient aspects of the
   structure and so I have found it necessary to
discard these theories and continue the search
for a rule system capable of describing the
harmonic core root structure of the Chorale
type pitch class simultaneity representation.

Short Bibliography
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12-fold and microtonal pitch systems; Computer
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